Inflation beyond slow-roll and non-Gaussianity

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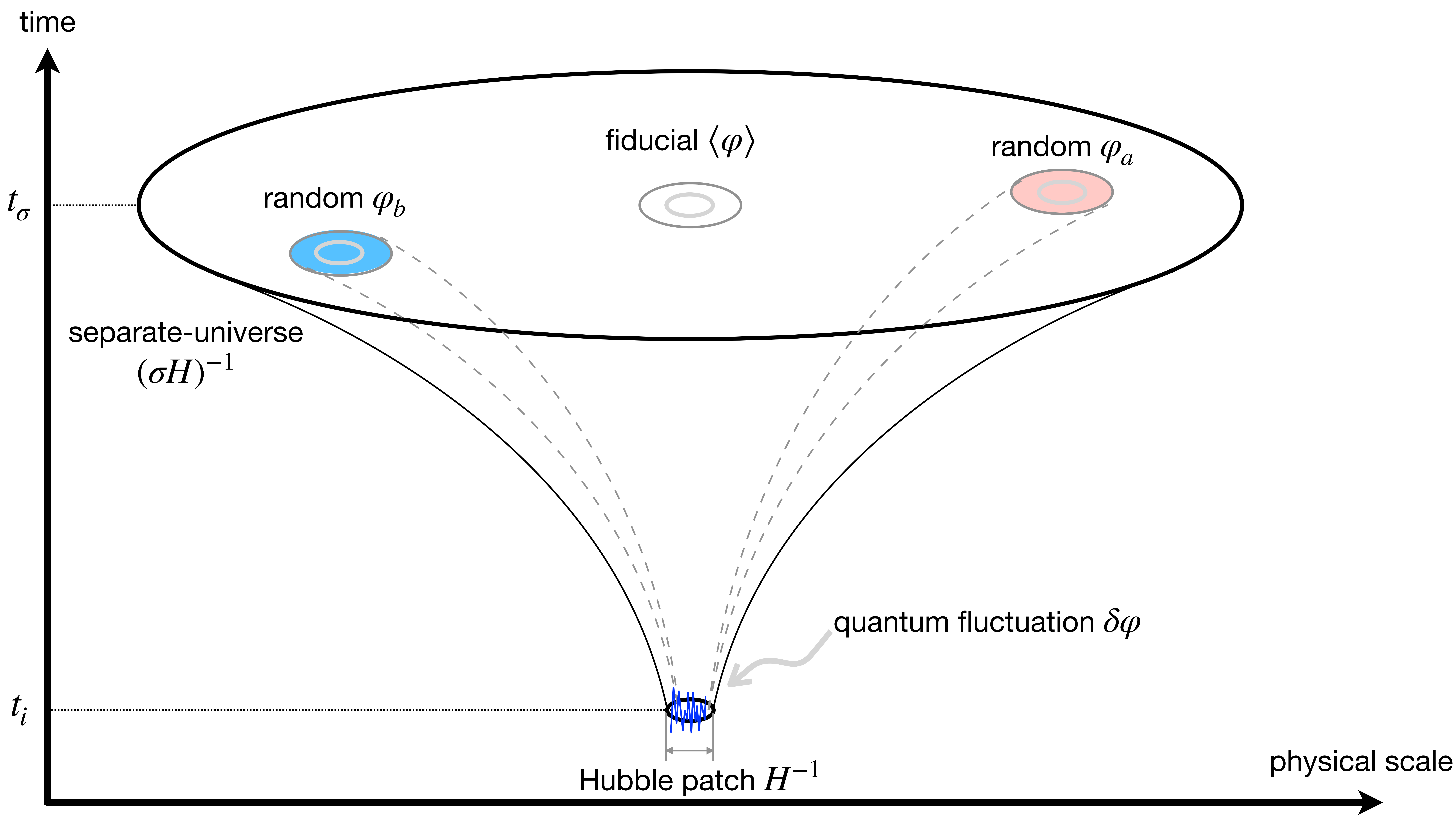
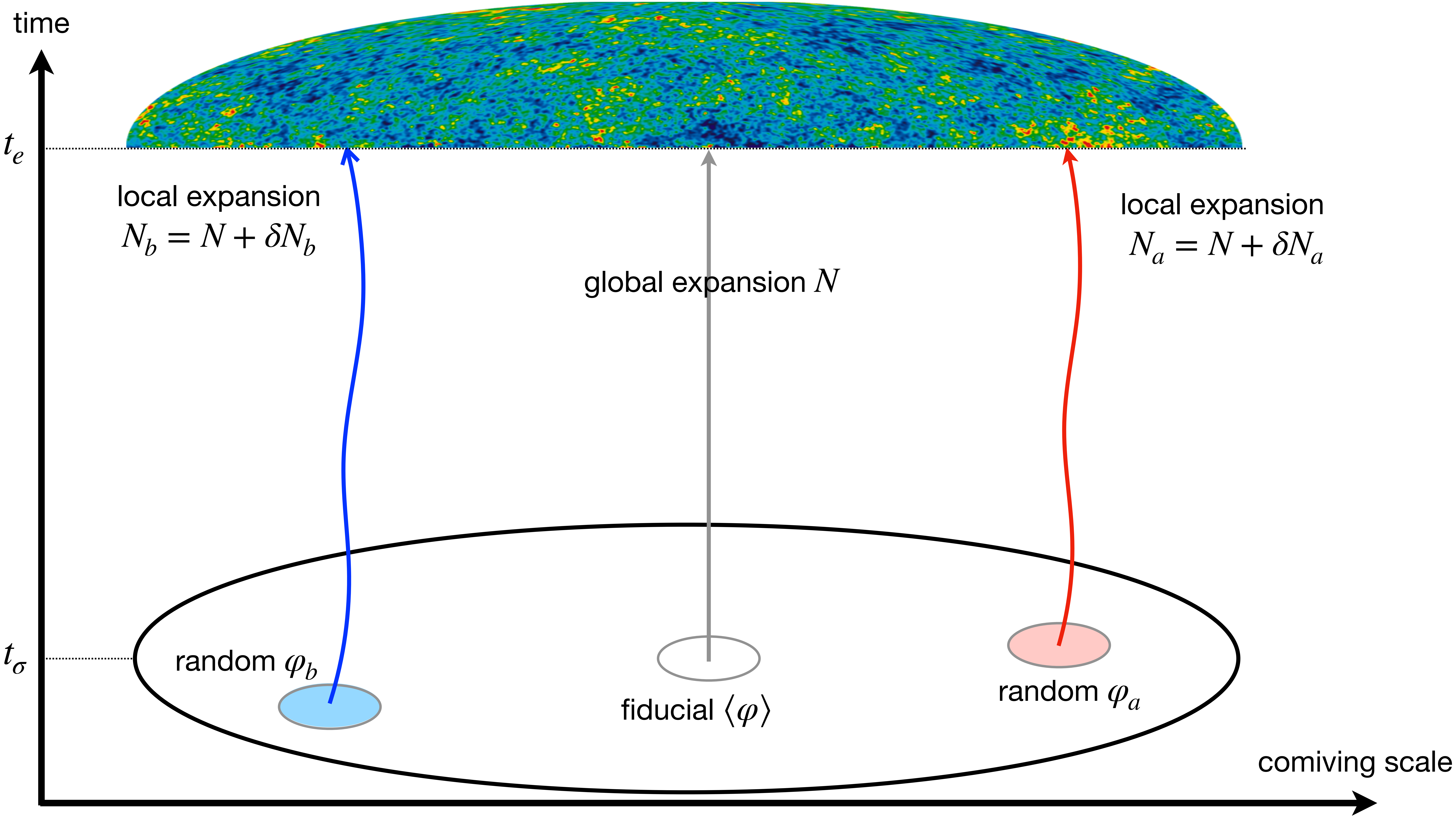
Cosmology Beyond the Analytic Lamppost (CoBALt)
Universite Paris-Sacley, June 26th, 2025

CONTENT

- Introduction: δN formalism
- Application: USR and general single-field
- Recent topics: gradient expansion and bubbles
- Conclusion

Angular scale
Planck
ACT
SPT
CMB anisotropy
Multipole moment ℓ
$\mathscr{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$
Reconstruction
Power spectrum of $\mathscr{R}$
$\sim 10^{-9}$
Gaussian
adiabatic
k/Mpc^{-1}
$\sim 10^{-2}$
USR,
multifield,
curvaton
Gaussian?
Adiabatic?
nonlinear
perturbation
gravitational
collapse
Scalar Perturbation
Induced GW
crosscheck
Primordial
Black Hole
PBH
Matarrese et al, PRD 47, 1311;
PRL 72, 320; PRD 58, 043504
Ananda et al, gr-qc/0612013
Bauman et al, hep-th/0703290
Zeldovich & Novikov 1966
Hawking 1971
Carr & Hawking 1974

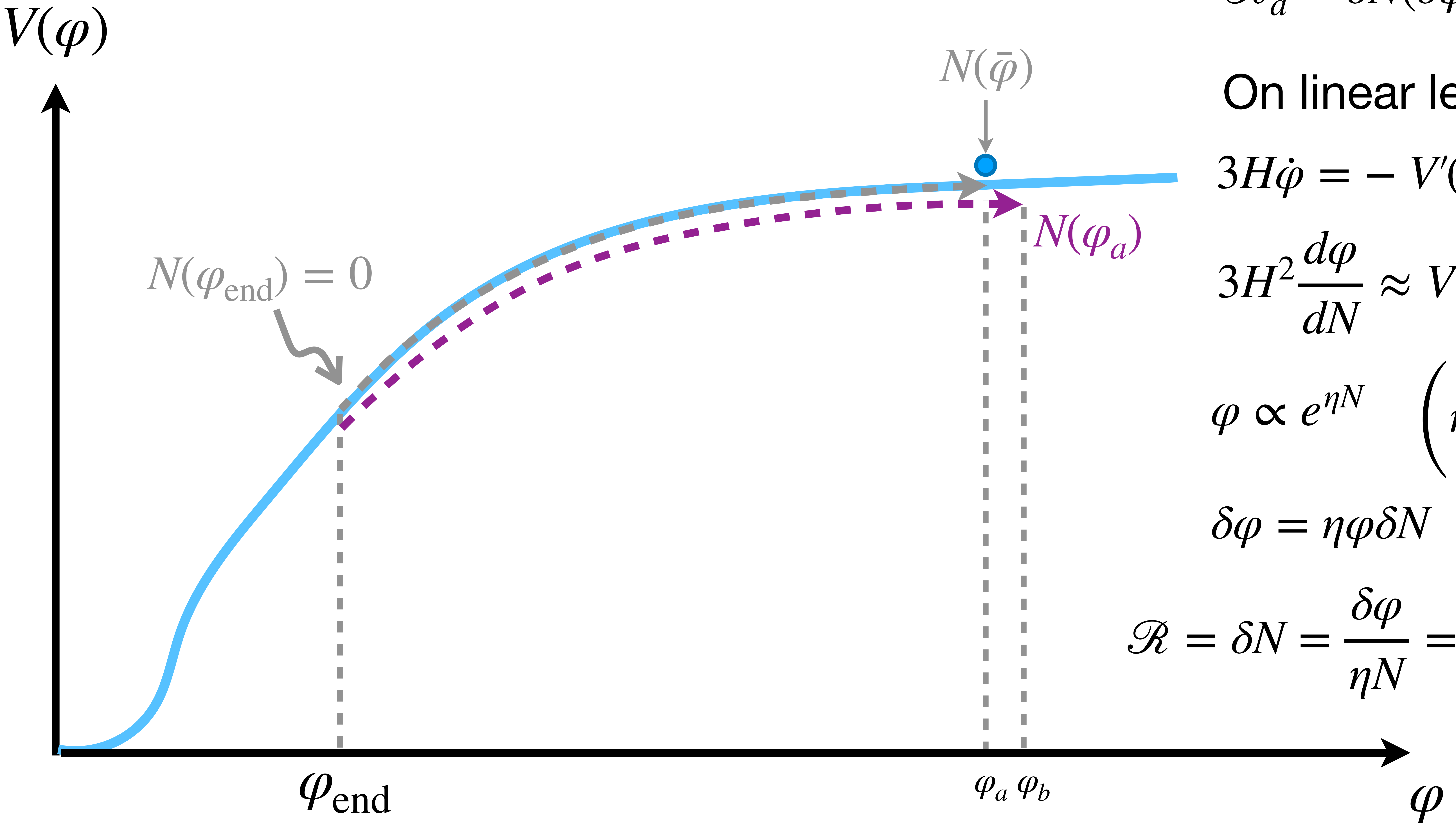

time
t_σ
t_i
fiducial $\langle\varphi\rangle$
random φ_a
random φ_b
separate-universe
$(\sigma H)^{-1}$
quantum fluctuation $\delta\varphi$
Hubble patch H^{-1}
physical scale



time
t_e
t_σ
local expansion
$N_b = N + \delta N_b$
global expansion N
local expansion
$N_a = N + \delta N_a$
random φ_b
fiducial $\langle\varphi\rangle$
random φ_a
comiving scale

$$ds^2 \sim a^2 e^{2\mathcal{R}} d\mathbf{x}^2$$

$$\mathcal{R}_a = \delta N(\delta\varphi_a)$$



On linear level:

$$3H\dot{\varphi} = -V'(\varphi)$$

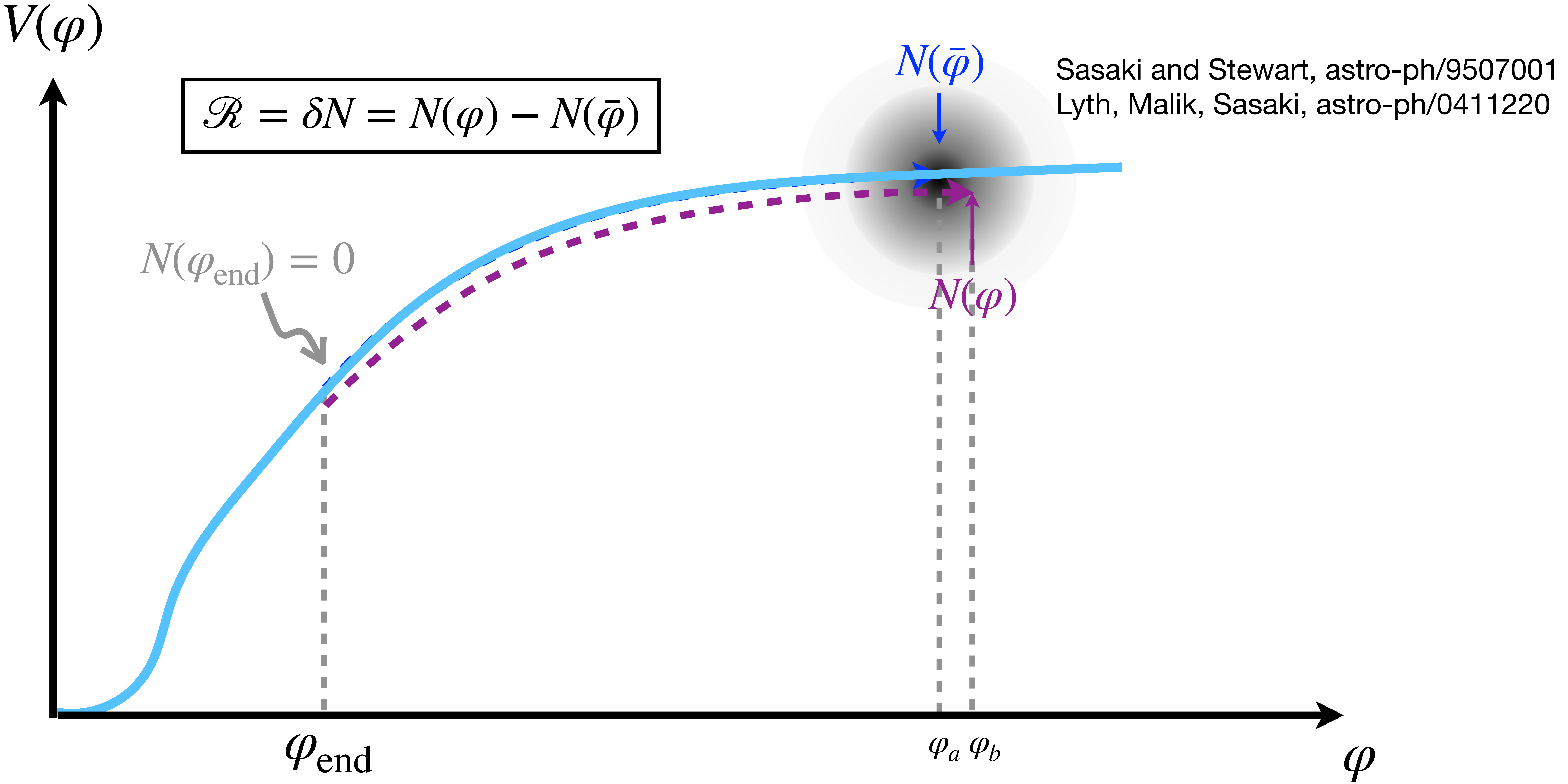
$$3H^2\frac{d\varphi}{dN} \approx V''(\varphi)\varphi$$

$$\varphi \propto e^{\eta N} \quad \left(\eta \equiv \frac{V''}{3H^2}\right)$$

$$\delta\varphi = \eta\varphi\delta N$$

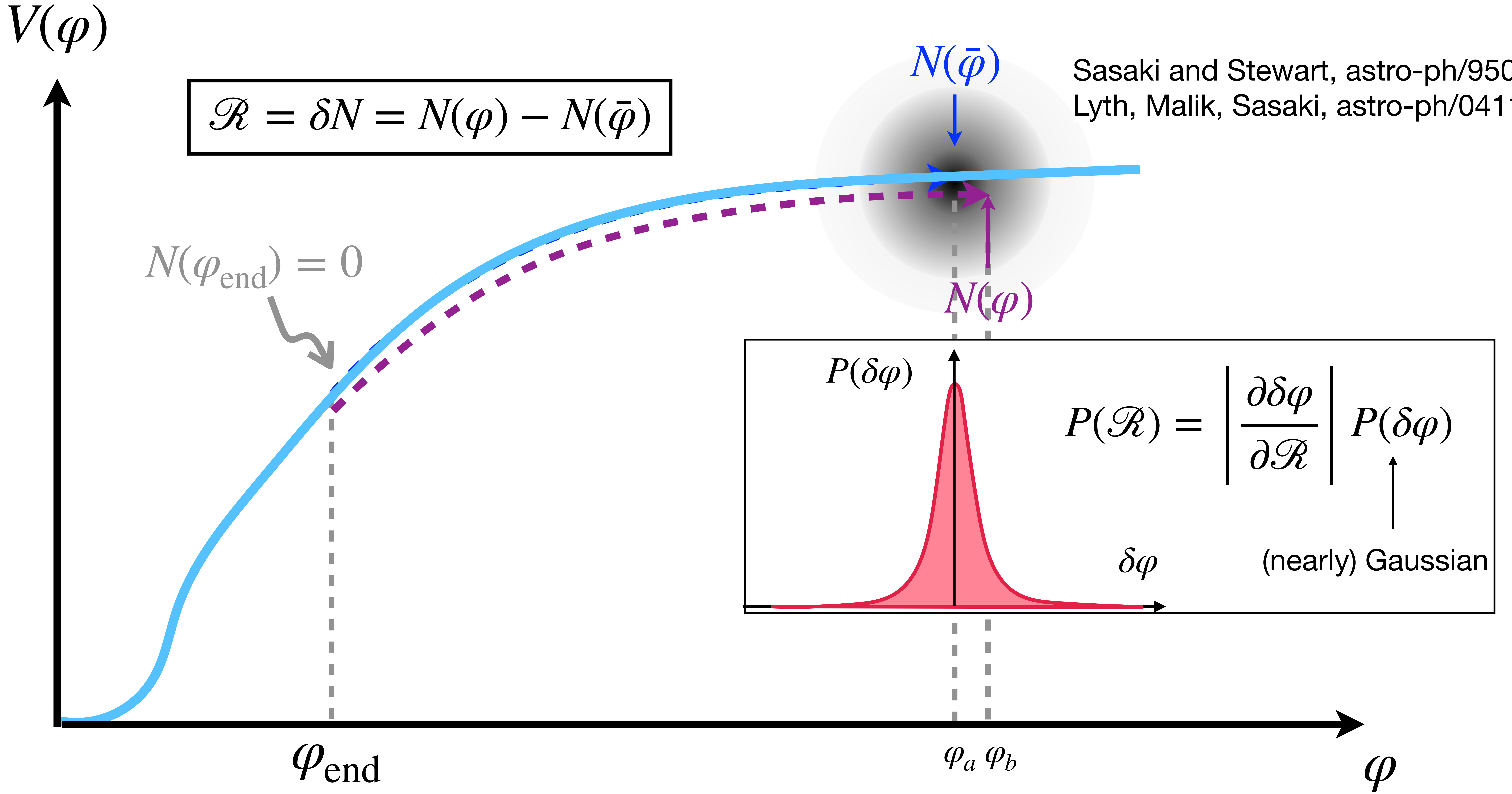
$$\mathcal{R} = \delta N = \frac{\delta\varphi}{\eta N} = -H\frac{\delta\varphi}{\dot{\varphi}}$$

δN formalism



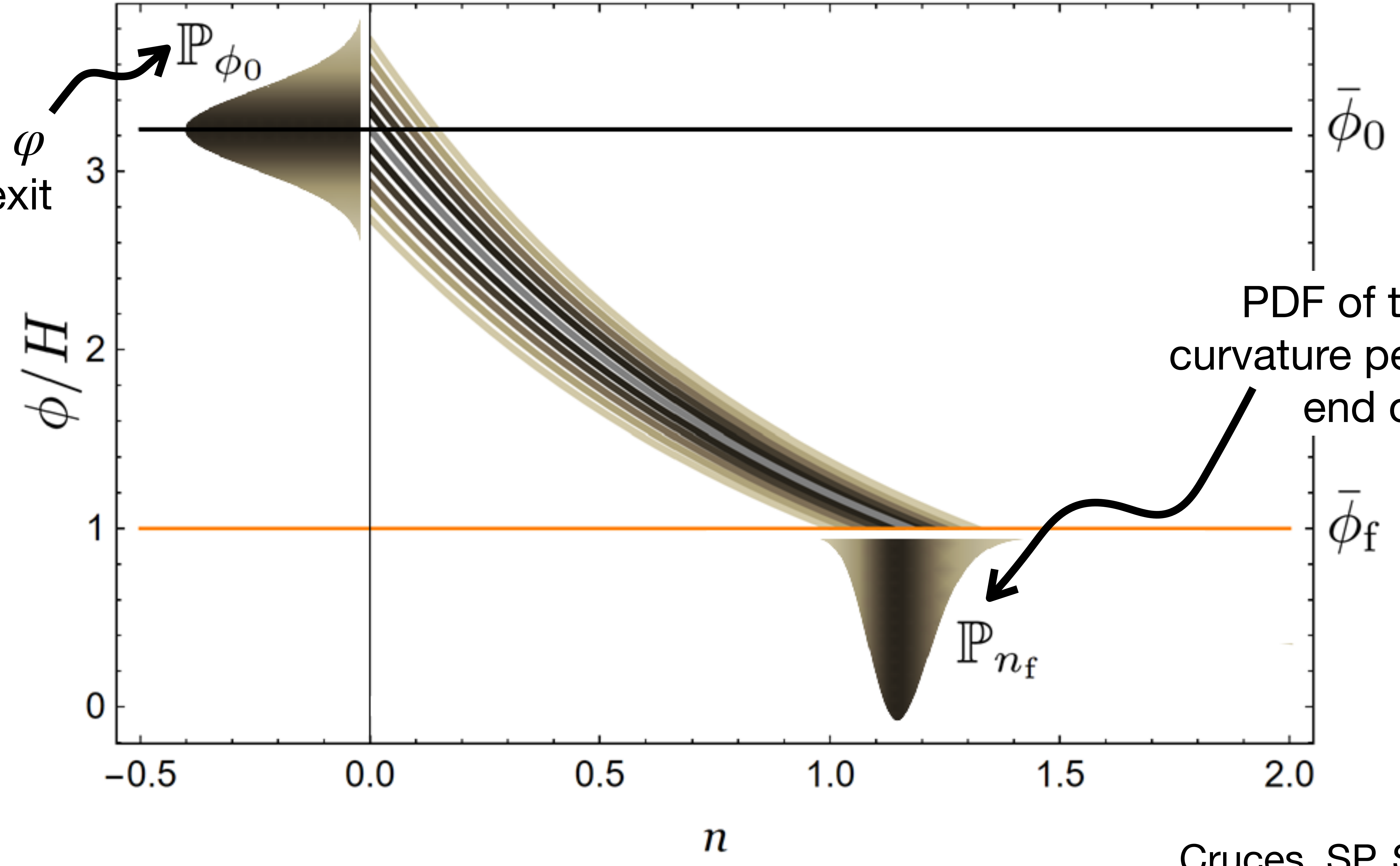
δN formalism

Sasaki and Stewart, astro-ph/9507001
Lyth, Malik, Sasaki, astro-ph/0411220



δN formalism

PDF of inflaton φ
at the horizon exit



PDF of the comoving
curvature perturbation at the
end of inflation

Cruces, SP, Sasaki, 2505.24590

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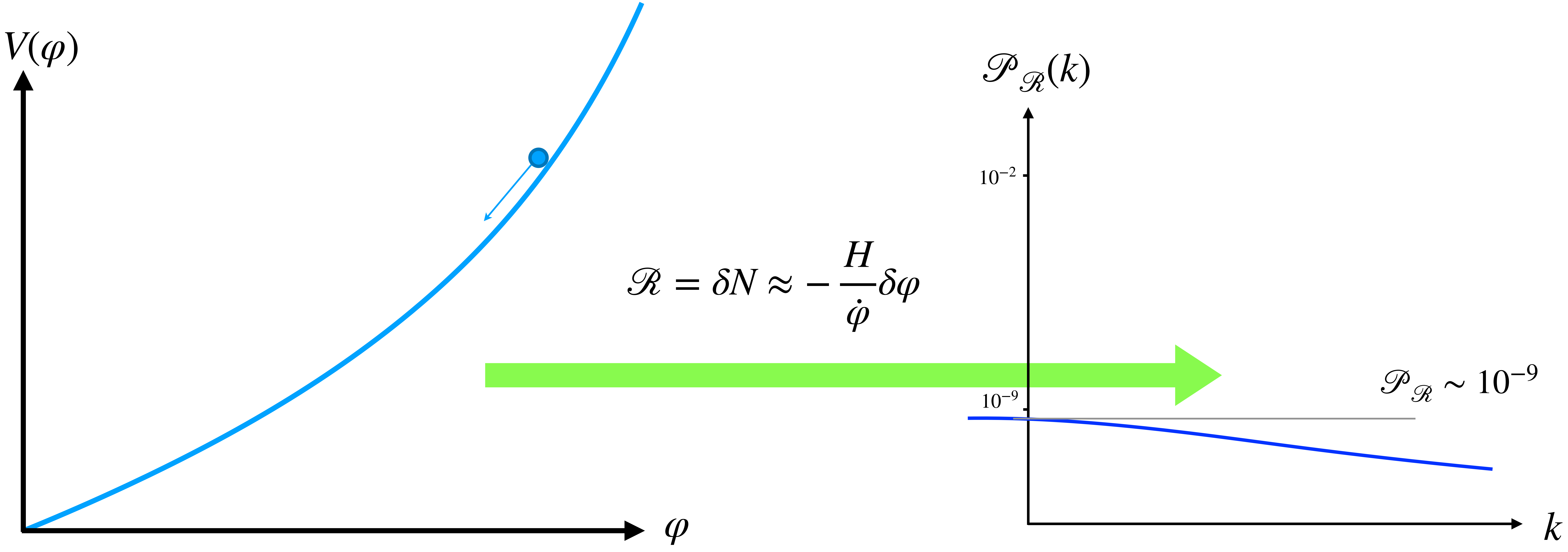
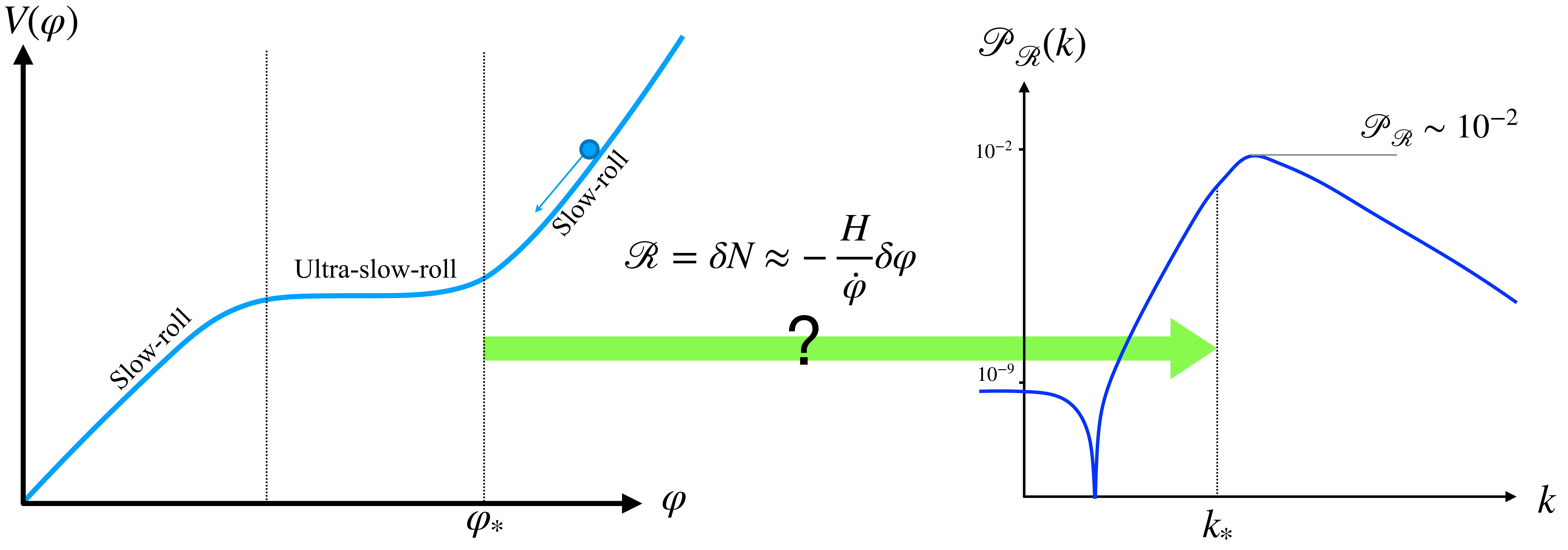
Gaussian Curvature Perturbation



$$\mathscr{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$$

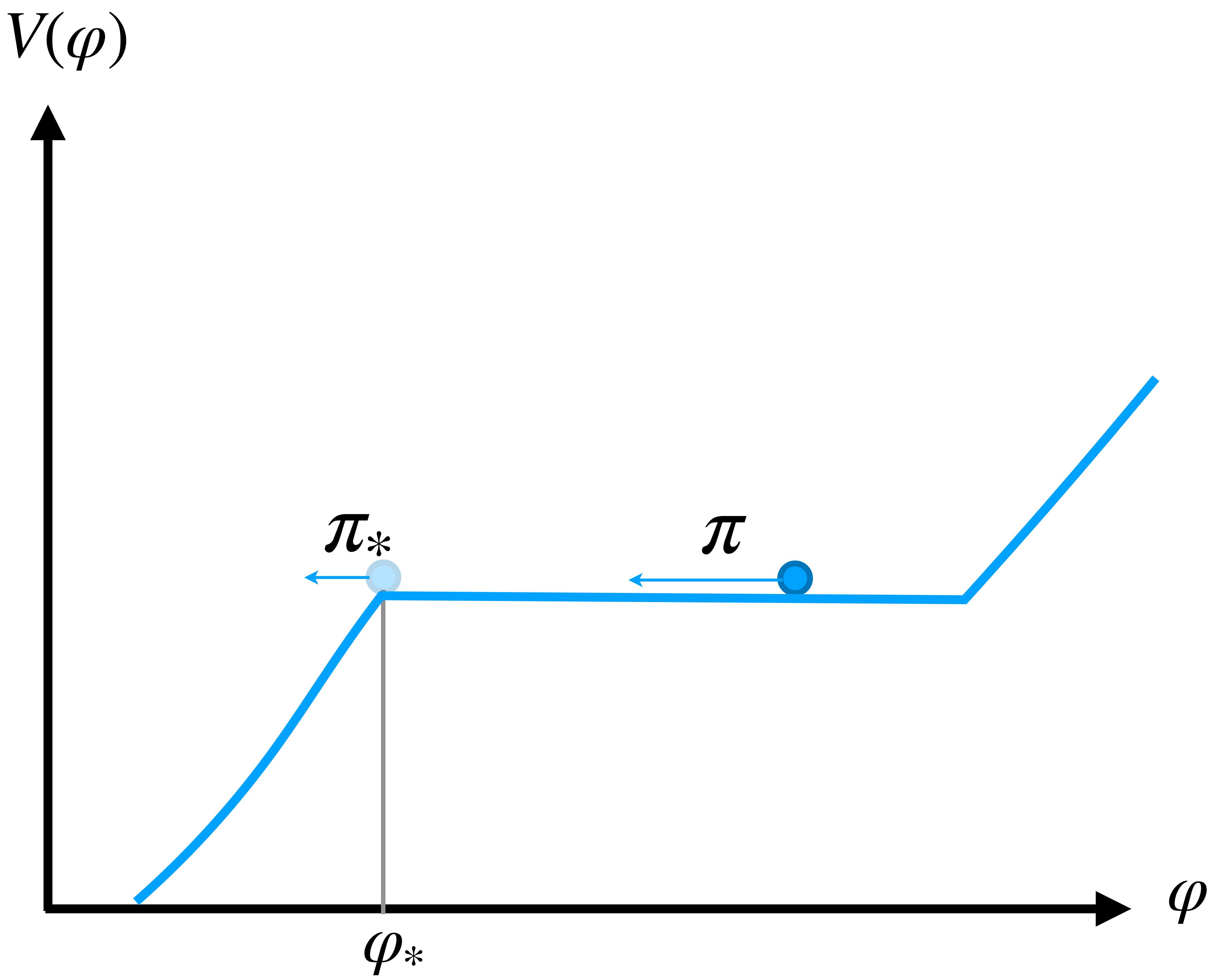
Stewart and Sasaki, astro-ph/9507001
Lyth and Roquigez, astro-ph/0504045

Ultra-slow-roll Inflation



Starobinski, JETP Lett. 55, 489
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP & Jianing Wang, 2209.14183

Ultra-slow-roll inflation



$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}N^2} - 3\frac{\mathrm{d}\varphi}{\mathrm{d}N} = 0 \qquad N = \int_{t_*}^{t} H\mathrm{d}t$$

$$\varphi(N) = \varphi_* + \frac{\pi_*}{3}\left(1 - e^{3N}\right)$$

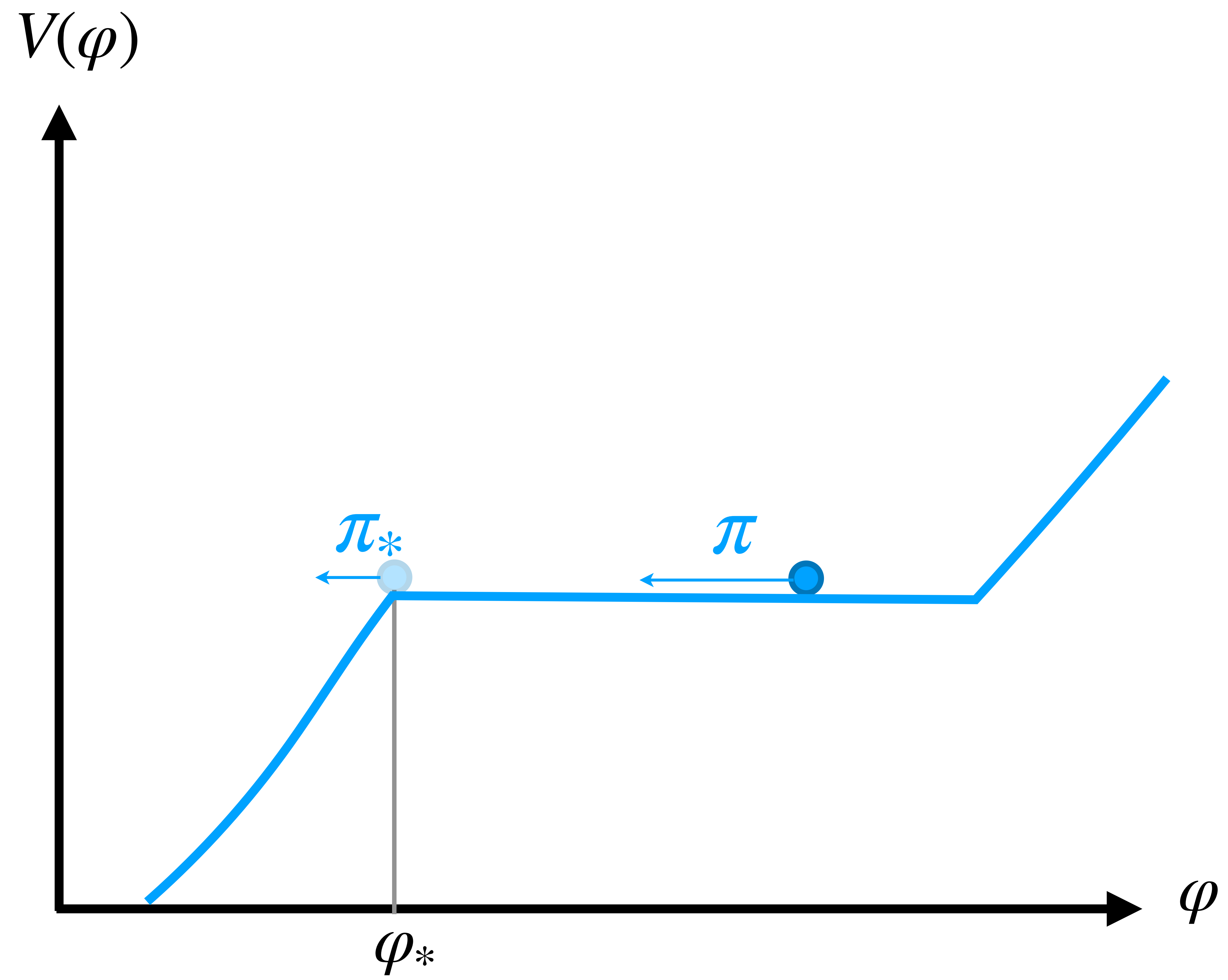
$$\pi(N) \equiv -\frac{\mathrm{d}\varphi}{\mathrm{d}N} = \pi_* e^{3N}$$

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

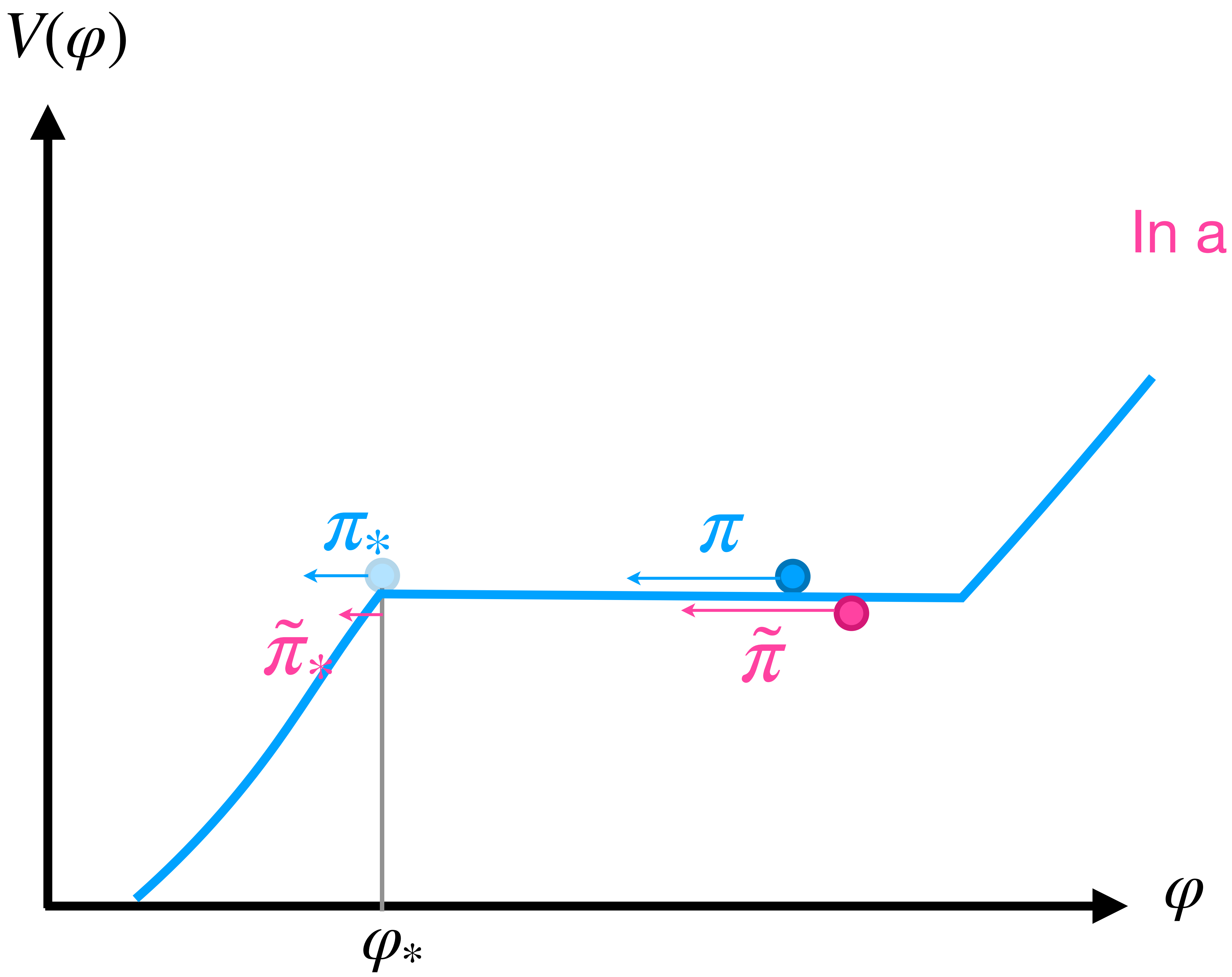
Ultra-slow-roll inflation

In the “fiducial” patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$



Ultra-slow-roll inflation



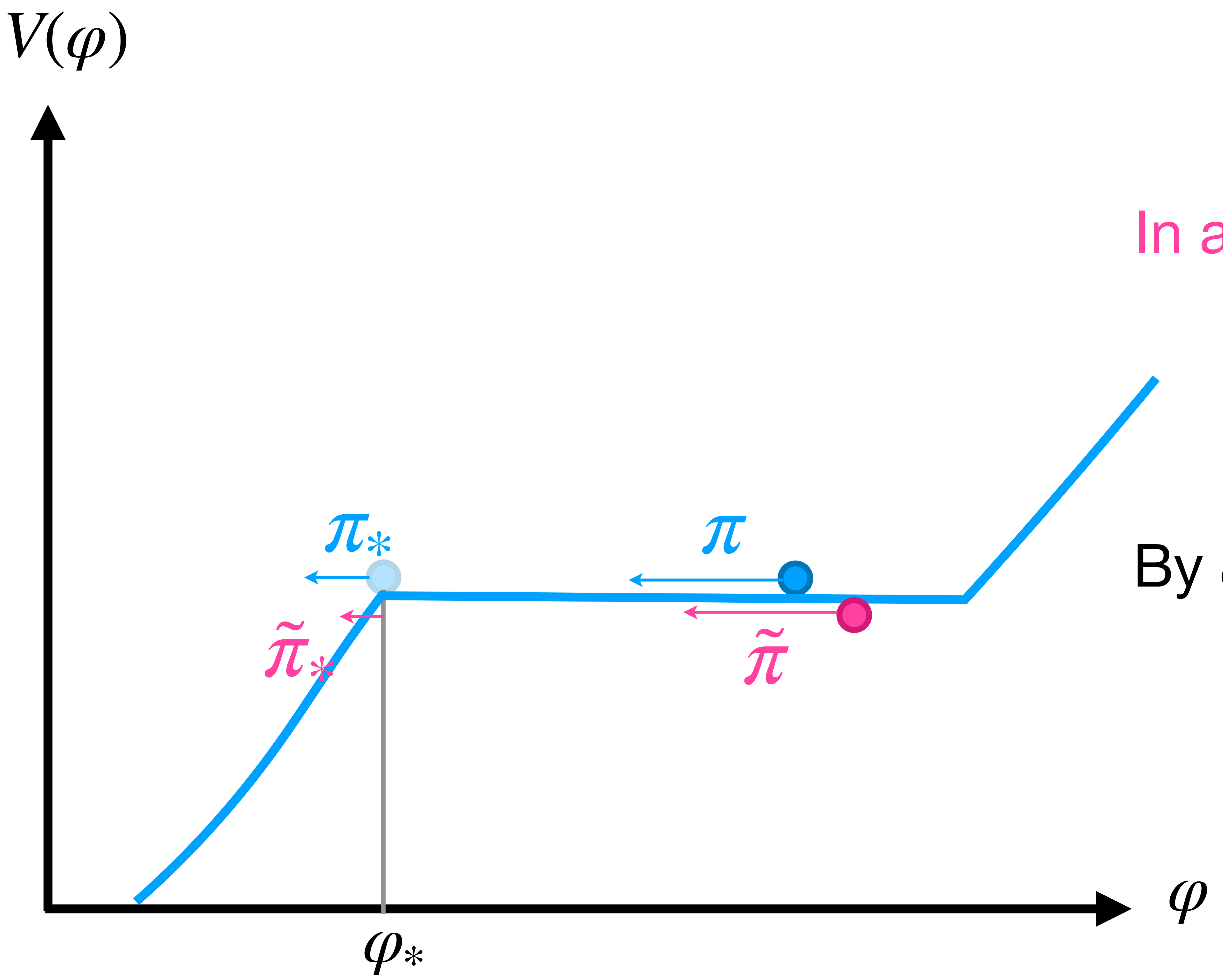
In the "fiducial" patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

In a perturbed patch

$$\tilde{N} = -\frac{1}{3}\ln\frac{\tilde{\pi}_*}{\tilde{\pi}}$$

Ultra-slow-roll inflation



In the “fiducial” patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

In a perturbed patch

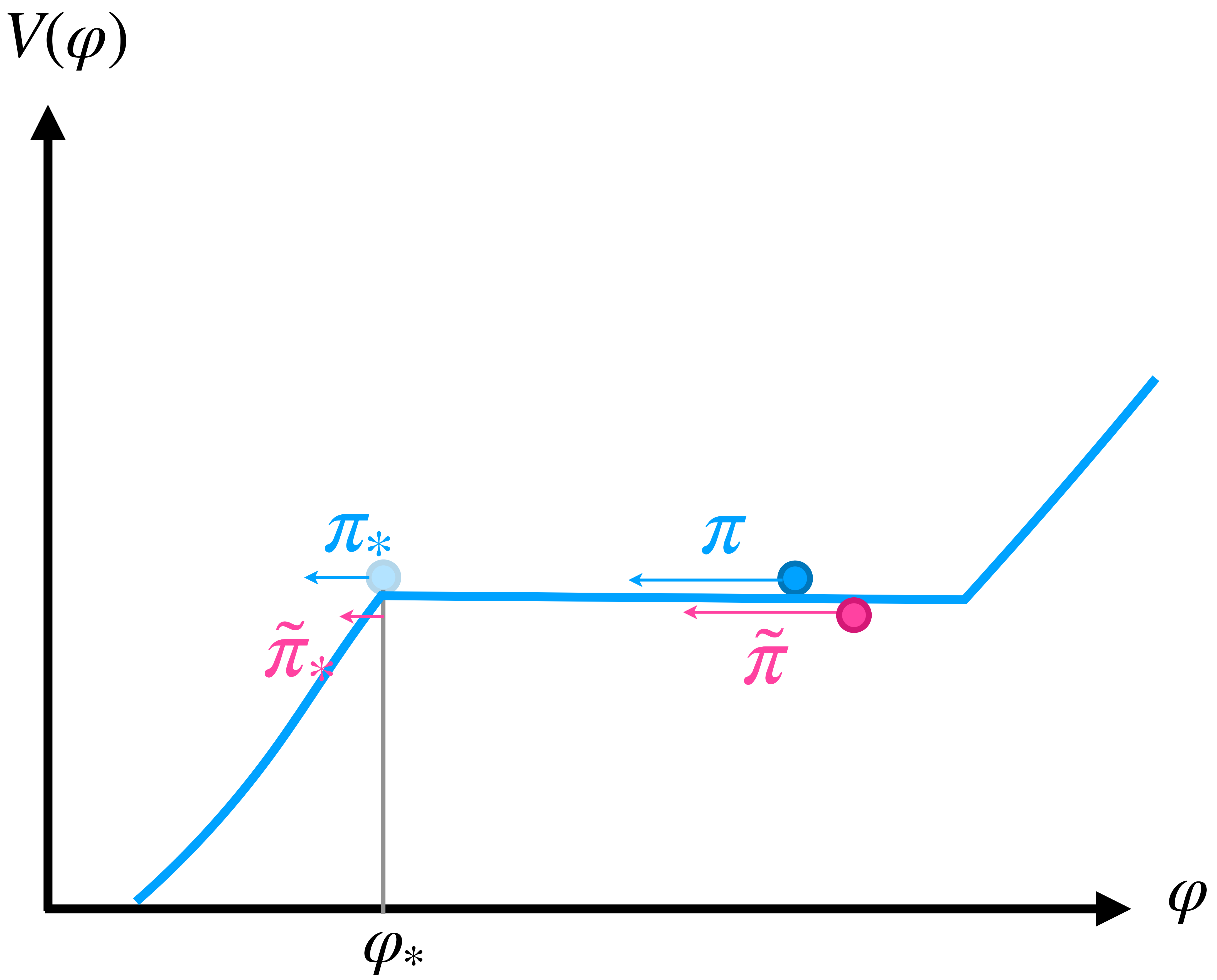
$$\tilde{N} = -\frac{1}{3}\ln\frac{\tilde{\pi}_*}{\tilde{\pi}}$$

By δN formalism, the curvature perturbation is

$$\mathscr{R} = \delta N = \tilde{N} - N = \frac{1}{3}\ln\frac{\tilde{\pi}}{\pi}\frac{\pi_*}{\tilde{\pi}_*}$$

$$= \frac{1}{3}\ln\left(1 - \frac{\delta\pi}{\pi}\right) - \frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll inflation



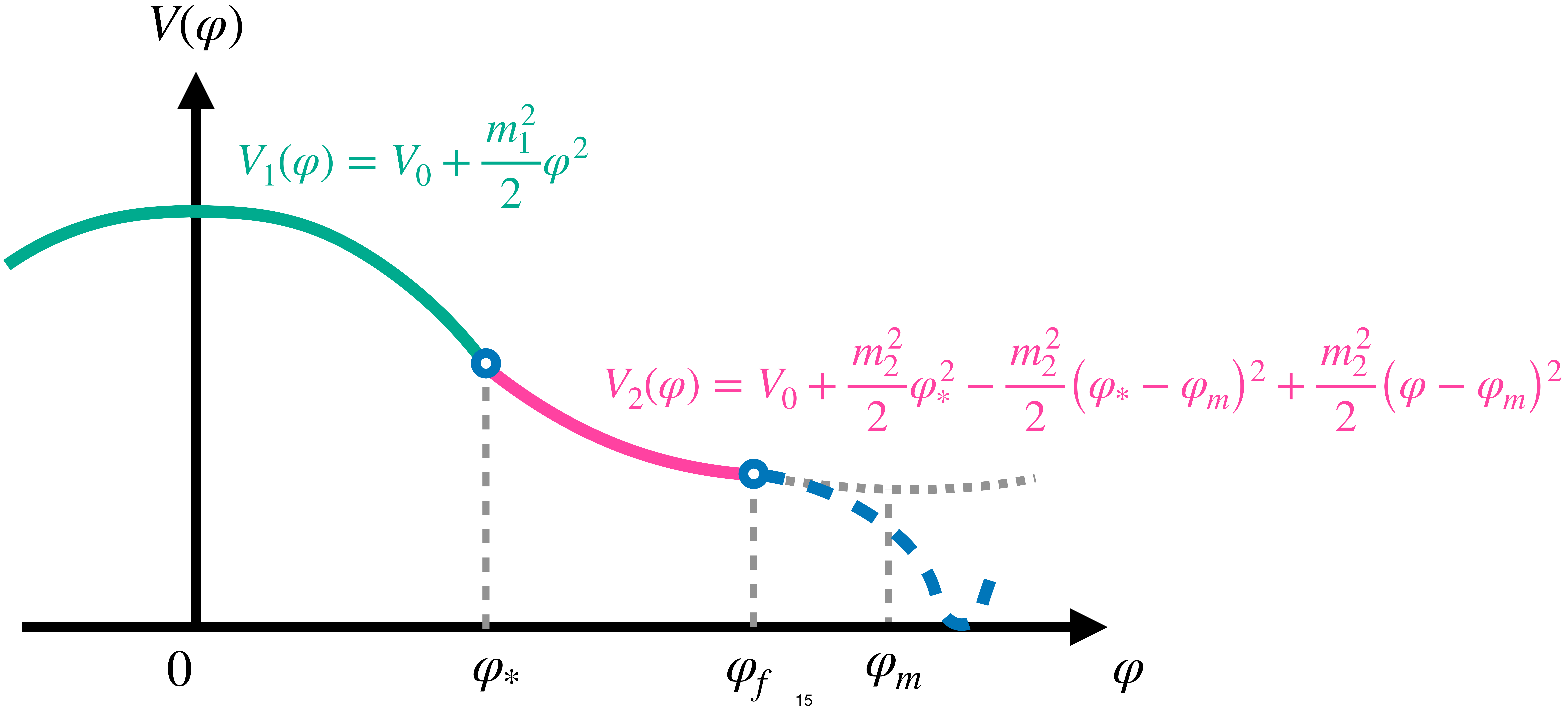
$$\mathscr{R} = -\frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

$$\left(f_{\rm NL} = \frac{5}{2},\quad g_{\rm NL} = -\frac{25}{3},\ \cdots\right)$$

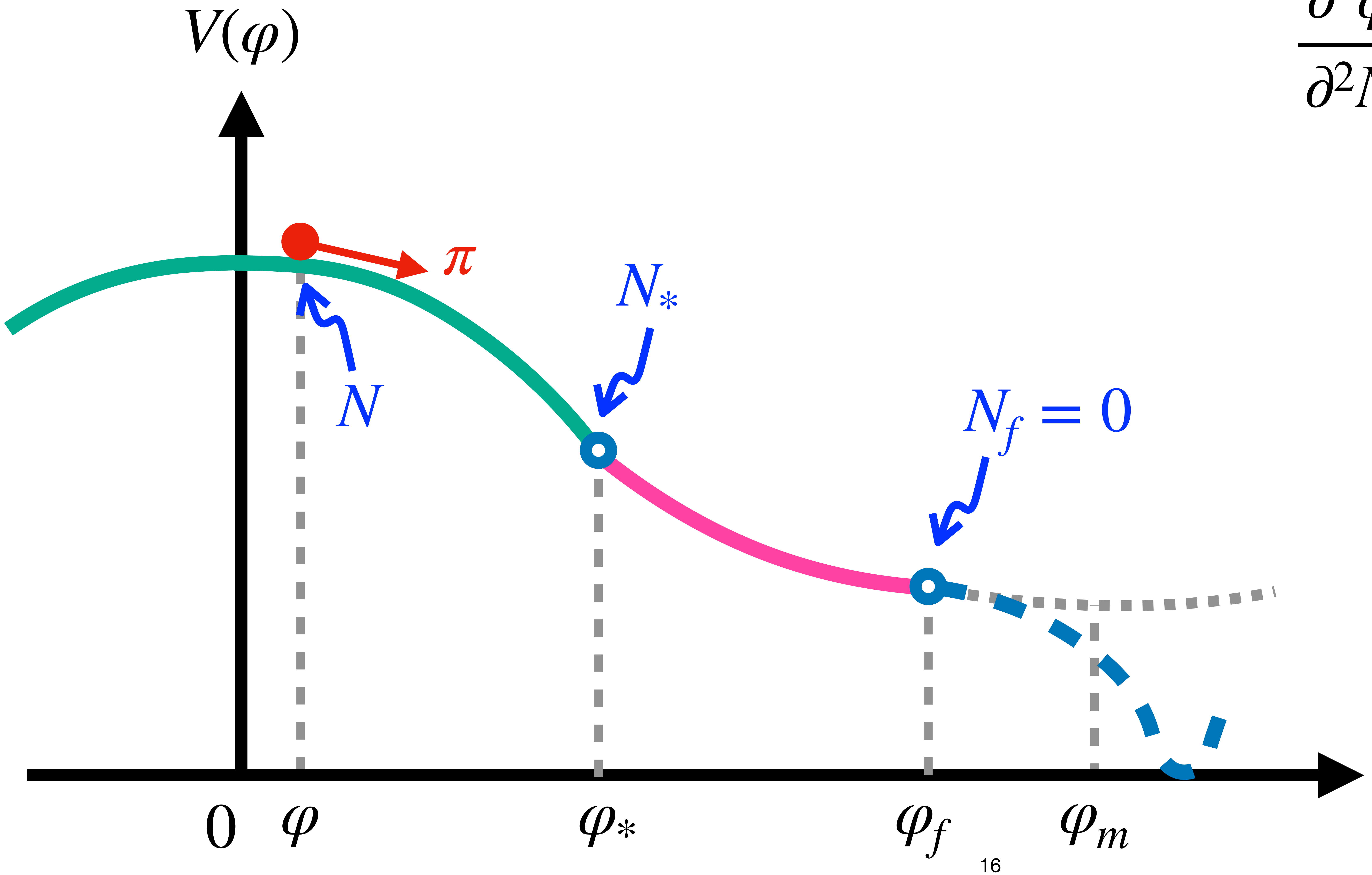
Namjoo, Firouzjahi, Sasaki, 1210.3692
Chen, Firouzjahi, Komatsu, Namjoo, Sasaki, 1308.5341
Cai, Chen, Namjoo, Sasaki, Wang, Wang, 1712.09998
Biagetti, Franciolini, Kehagias, Riotto, 1804.07124
Passaglia, Hu, Motohashi, 1812.08243
SP and Sasaki, 2211.13932
SP, 2404.06151
Relation with stochastic approach, see e.g.
Jackson et al 2410.13683, Cruces et al 2410.17987

Guillermo and Alejandro's talk

piecewise quadratic potential



piecewise quadratic potential



$$\frac{\partial^2\varphi}{\partial^2 N} - 3\frac{\partial\varphi}{\partial N} + 3\eta_V\varphi = 0$$

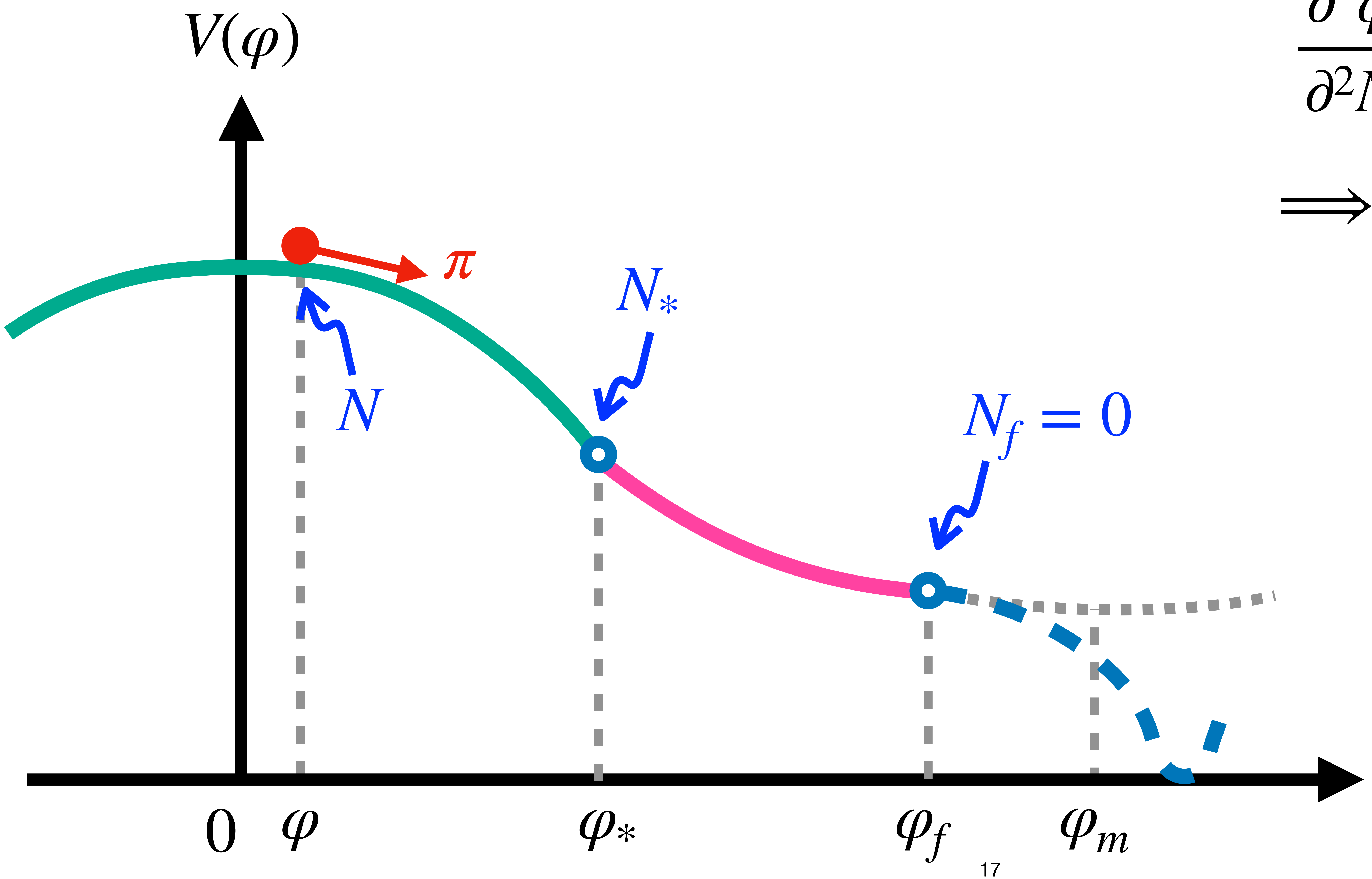
$$\eta_V = \frac{m_1^2}{3H^2}$$

$$N = \int_t^{t_f} H dt$$

$$V_1(\varphi) = V_0 + \frac{m_1^2}{2}\varphi^2$$

$$V_2(\varphi) = V_0 + \frac{m_2^2}{2}\varphi_*^2 - \frac{m_2^2}{2}(\varphi_* - \varphi_m)^2 + \frac{m_2^2}{2}(\varphi - \varphi_m)^2$$

background solution



$$\frac{\partial^2\varphi}{\partial^2 N} - 3\frac{\partial\varphi}{\partial N} + 3\eta_V\varphi = 0$$

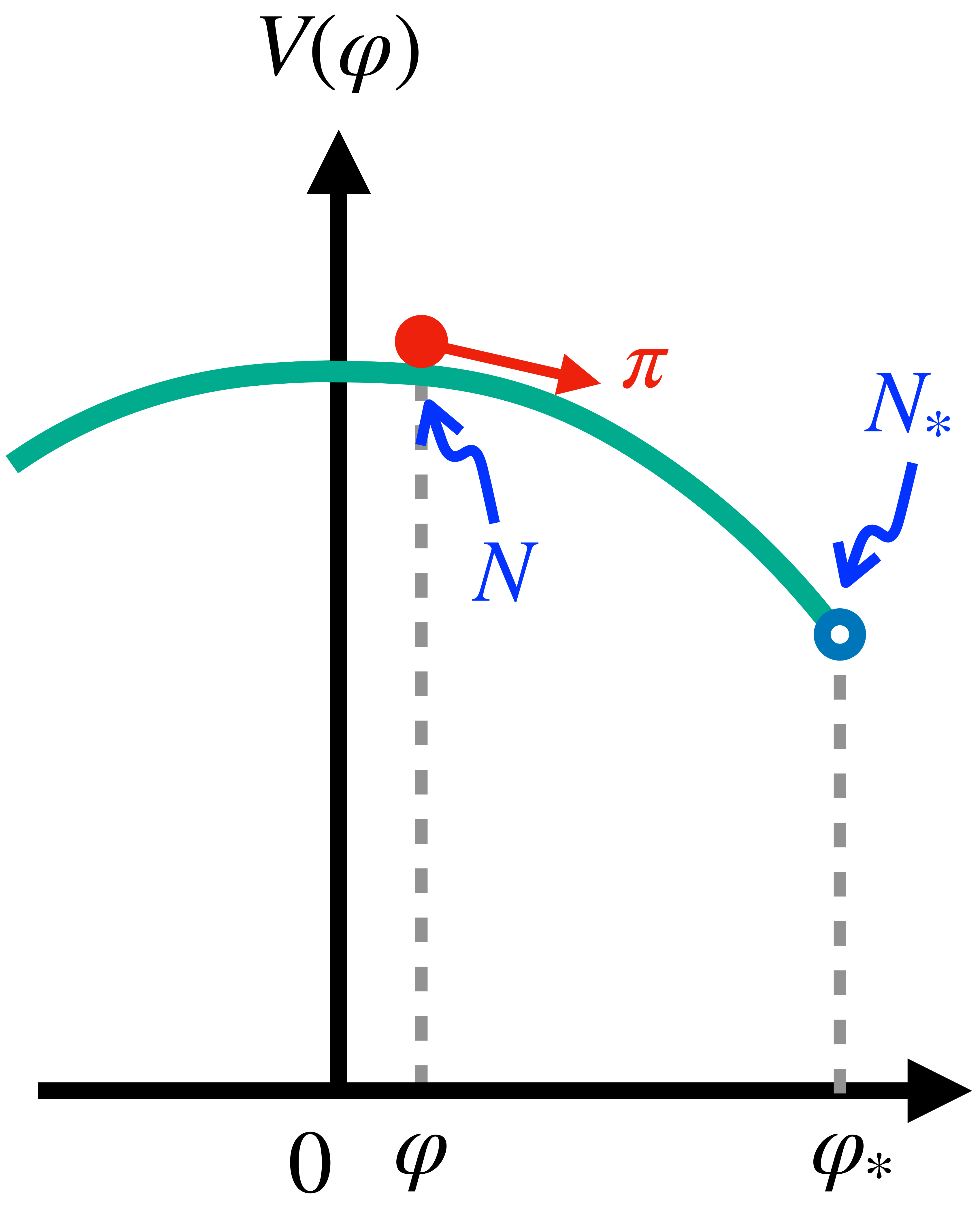
$$\Longrightarrow \quad \varphi = c_+e^{\lambda_+N} + c_-e^{\lambda_-N}$$

$$\lambda_\pm = \frac{3 \pm \sqrt{9 - 12\eta_V}}{2}$$

$$V_1(\varphi) = V_0 + \frac{m_1^2}{2}\varphi^2$$

$$V_2(\varphi) = V_0 + \frac{m_2^2}{2}\varphi_*^2 - \frac{m_2^2}{2}(\varphi_* - \varphi_m)^2 + \frac{m_2^2}{2}(\varphi - \varphi_m)^2$$

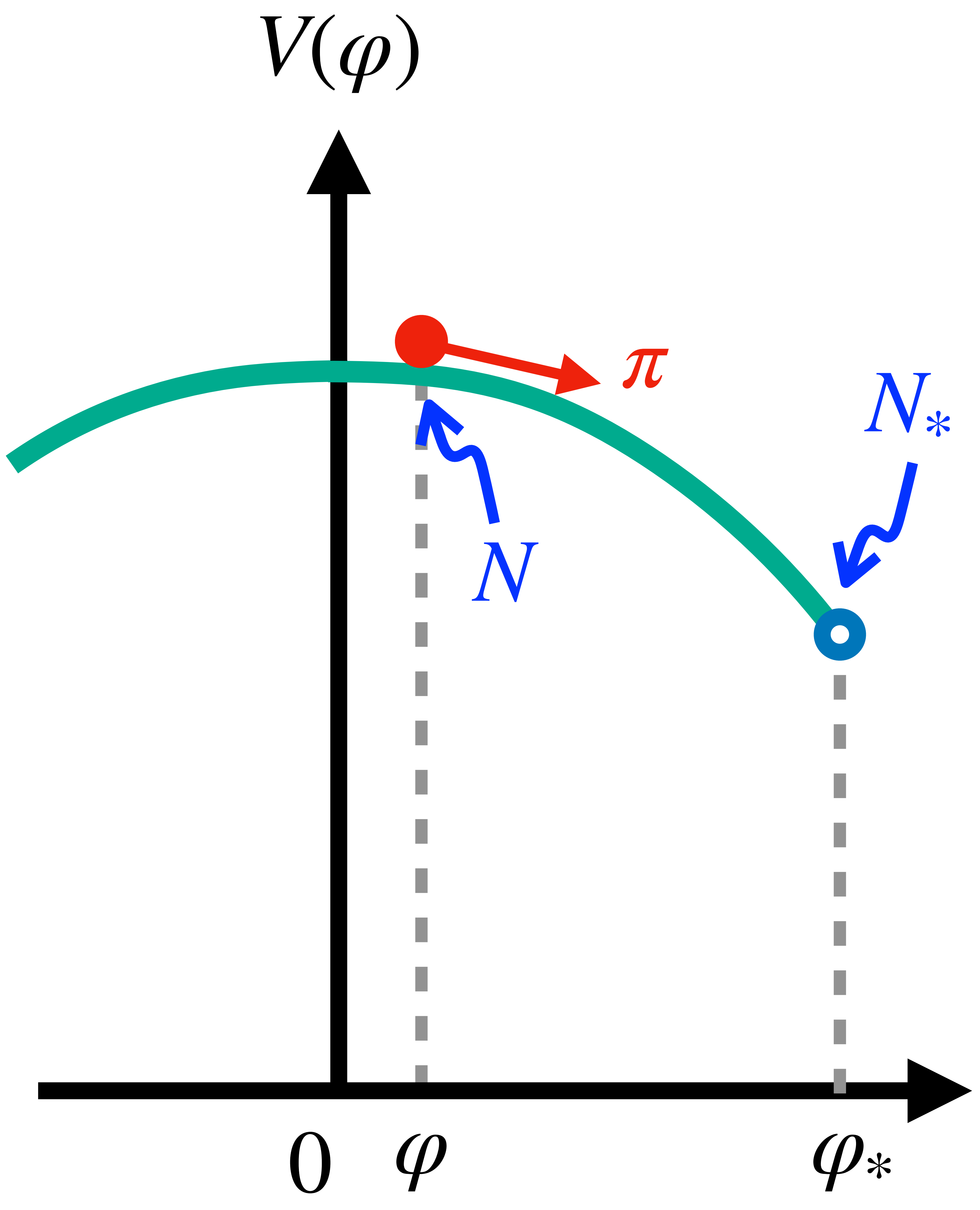
background solution



$$\varphi(N) = c_+ e^{\lambda_+(N-N_*)} + c_- e^{\lambda_-(N-N_*)}$$

$$-\pi(N) \equiv \frac{\partial \varphi}{\partial N} = \lambda_+ c_+ e^{\lambda_+(N-N_*)} + \lambda_- c_- e^{\lambda_-(N-N_*)}$$

background solution



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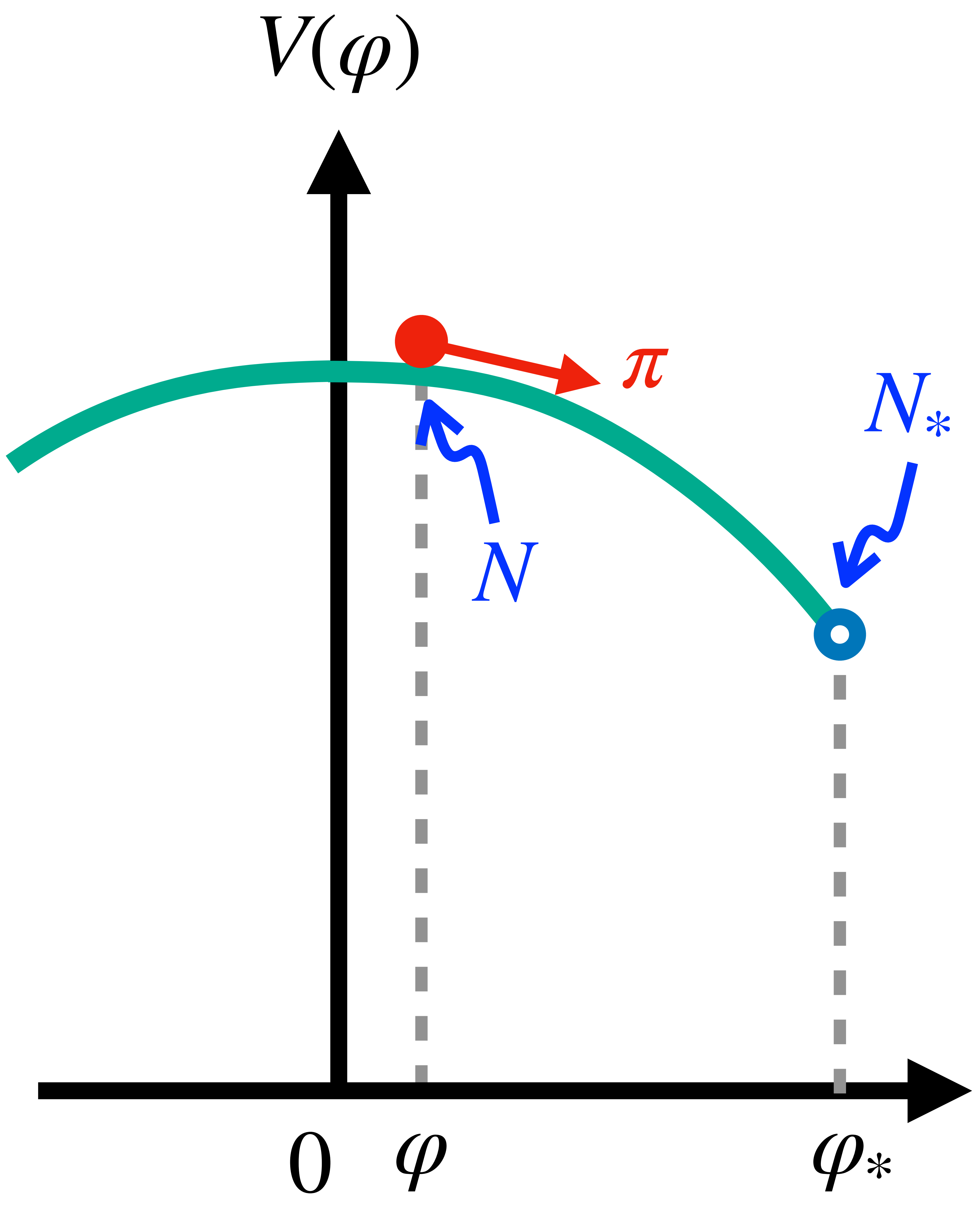
$$\varphi(N_*) \equiv \varphi_* = c_+ + c_-$$

$$-\pi(N_*) \equiv \pi_* = \lambda_+ c_+ + \lambda_- c_-$$

$$\Longrightarrow \quad c_\pm = \mp \frac{\pi_* + \lambda_\mp \varphi_*}{\lambda_+ - \lambda_-}$$

background solution



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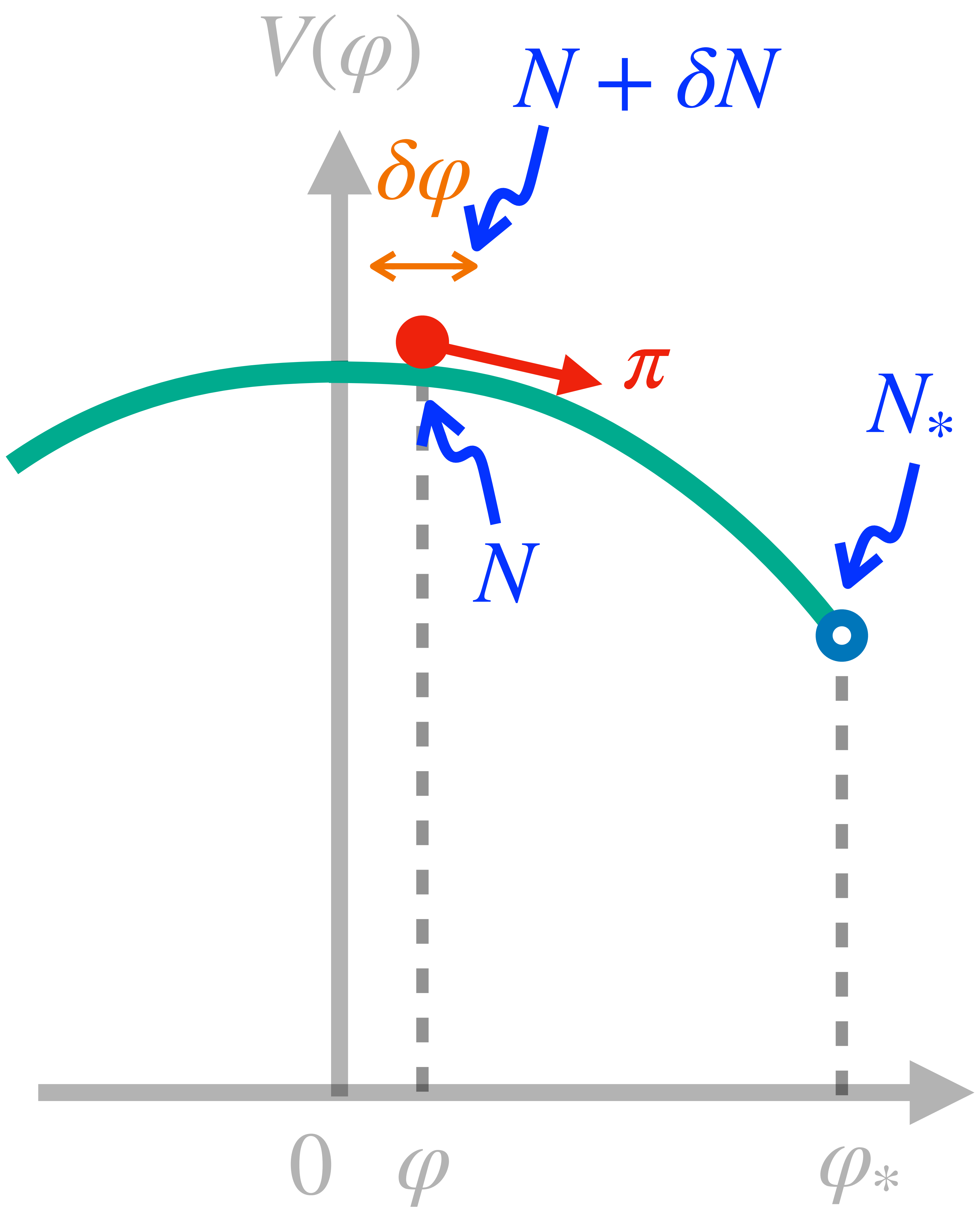
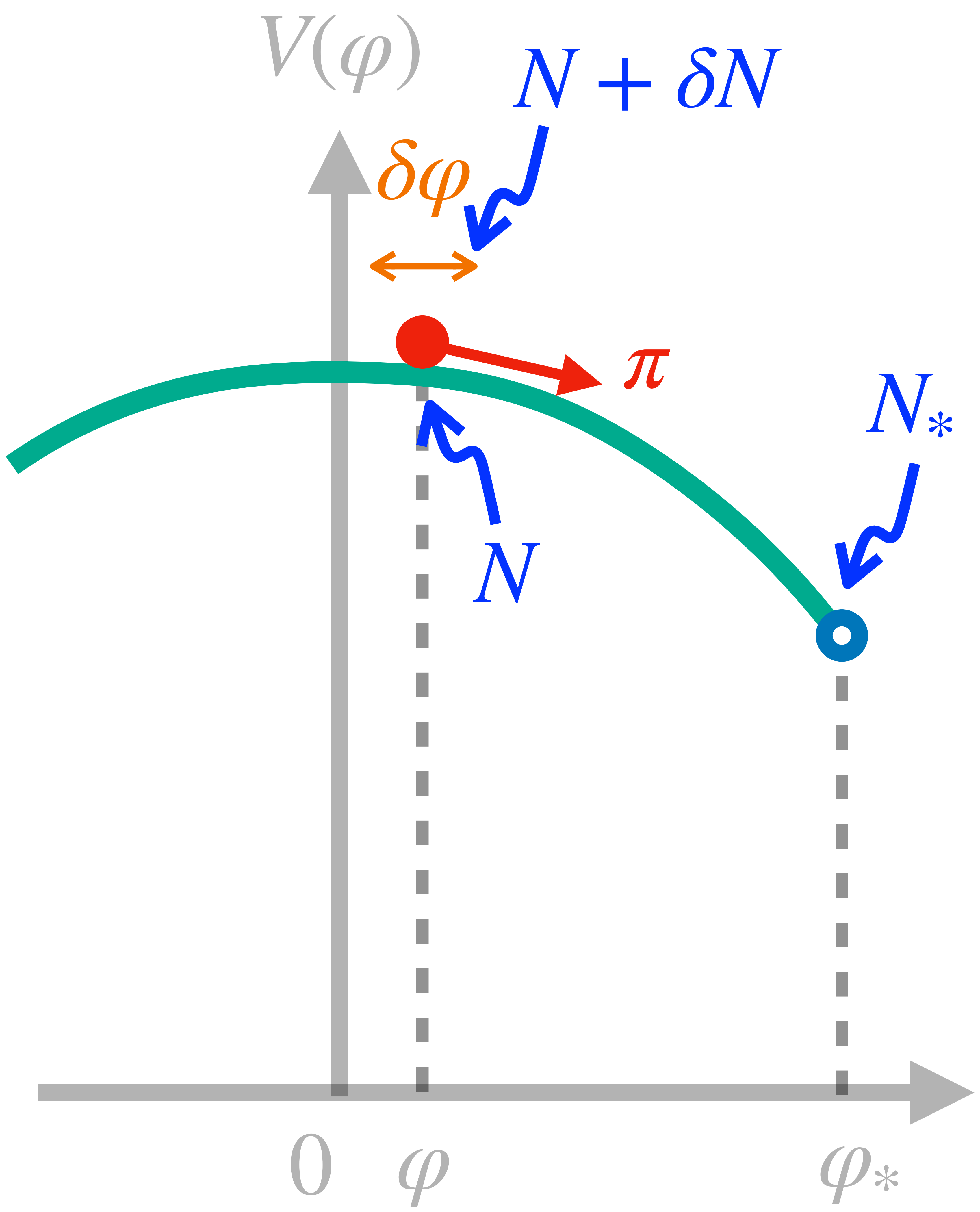
Logarithmic Duality



The (fiducial) e-folding number can be expressed by (φ, π) and their values on the boundary (φ_*, π_*).

$$\left.\begin{aligned} \frac{\pi + \lambda_+ \varphi}{\pi_* + \lambda_+ \varphi_*} &= e^{\lambda_+(N - N_*)} \\ \frac{\pi + \lambda_- \varphi}{\pi_* + \lambda_- \varphi_*} &= e^{\lambda_-(N - N_*)} \end{aligned}\right\} \Longrightarrow N - N_* = \frac{1}{\lambda_\pm} \ln \frac{\pi + \lambda_\mp \varphi}{\pi_* + \lambda_\mp \varphi_*}$$

Logarithmic Duality



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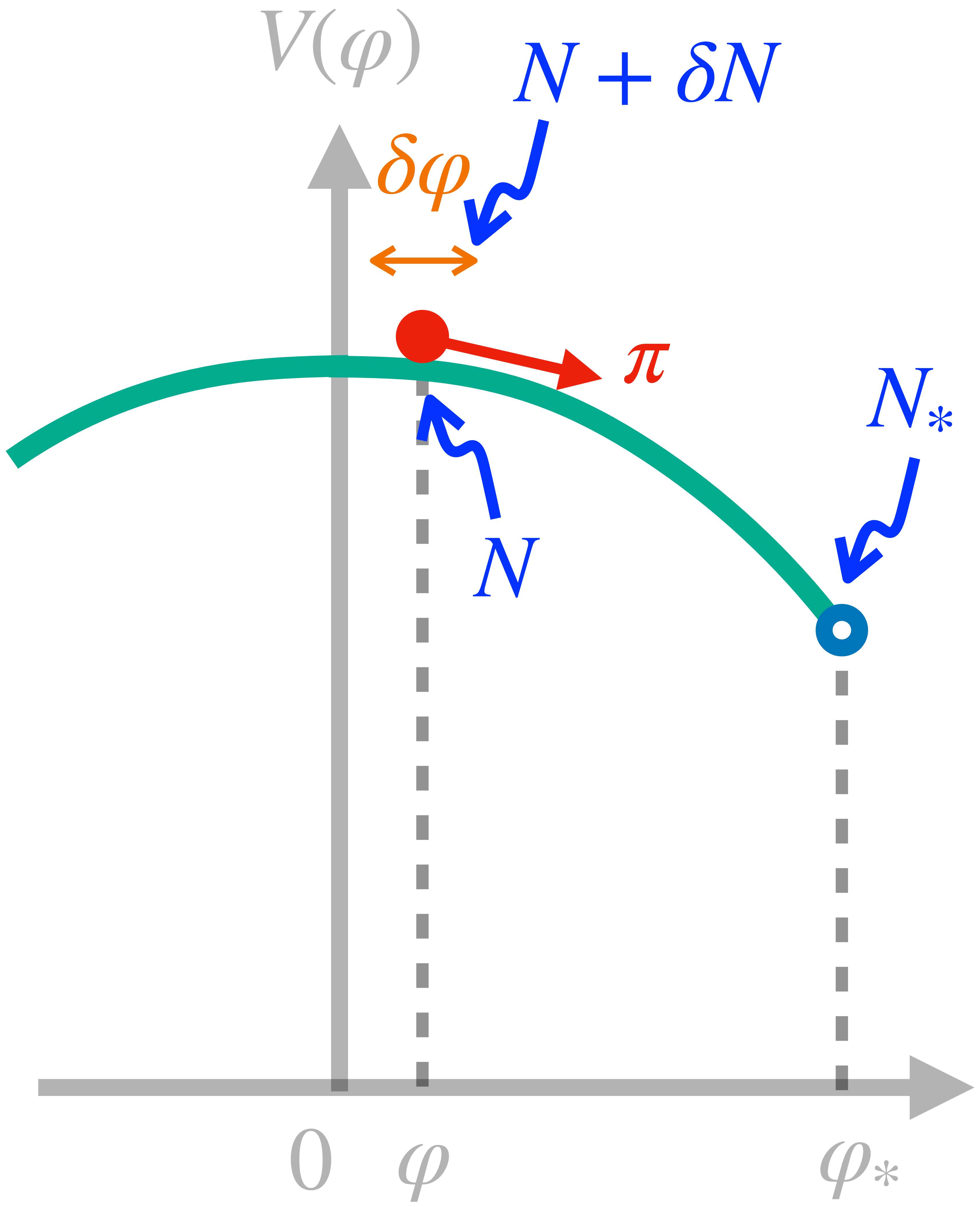
$$\left.\begin{aligned} \frac{\pi + \lambda_+ \varphi}{\pi_* + \lambda_+ \varphi_*} &= e^{\lambda_+(N-N_*)} \\ \frac{\pi + \lambda_- \varphi}{\pi_* + \lambda_- \varphi_*} &= e^{\lambda_-(N-N_*)} \end{aligned}\right\} \Longrightarrow N - N_* = \frac{1}{\lambda_\pm} \ln \frac{\pi + \lambda_\mp \varphi}{\pi_* + \lambda_\mp \varphi_*}$$

For another trajectory, we take the perturbation as

$$\left.\begin{aligned} N &\to N + \delta N \\ \varphi &\to \varphi + \delta\varphi \\ \pi &\to \pi + \delta\pi \\ \pi_* &\to \pi_* + \delta\pi_* \end{aligned}\right\} \quad N - N_* + \delta(N - N_*) = \frac{1}{\lambda_\pm} \ln \frac{\pi + \delta\pi + \lambda_\mp(\varphi + \delta\varphi)}{\pi_* + \delta\pi_* + \lambda_\mp \varphi_*}$$

And then subtract the fiducial N from $N + \delta N$:

Logarithmic Duality



$$\left.\begin{array}{l} \dfrac{\pi+\lambda_+\varphi}{\pi_*+\lambda_+\varphi_*} = e^{\lambda_+(N-N_*)} \\ \dfrac{\pi+\lambda_-\varphi}{\pi_*+\lambda_-\varphi_*} = e^{\lambda_-(N-N_*)} \end{array}\right\} \Longrightarrow N-N_* = \frac{1}{\lambda_\pm}\ln\frac{\pi+\lambda_\mp\varphi}{\pi_*+\lambda_\mp\varphi_*}$$

$$\Longrightarrow \mathscr{R} = \delta(N-N_*)$$

$$= \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$

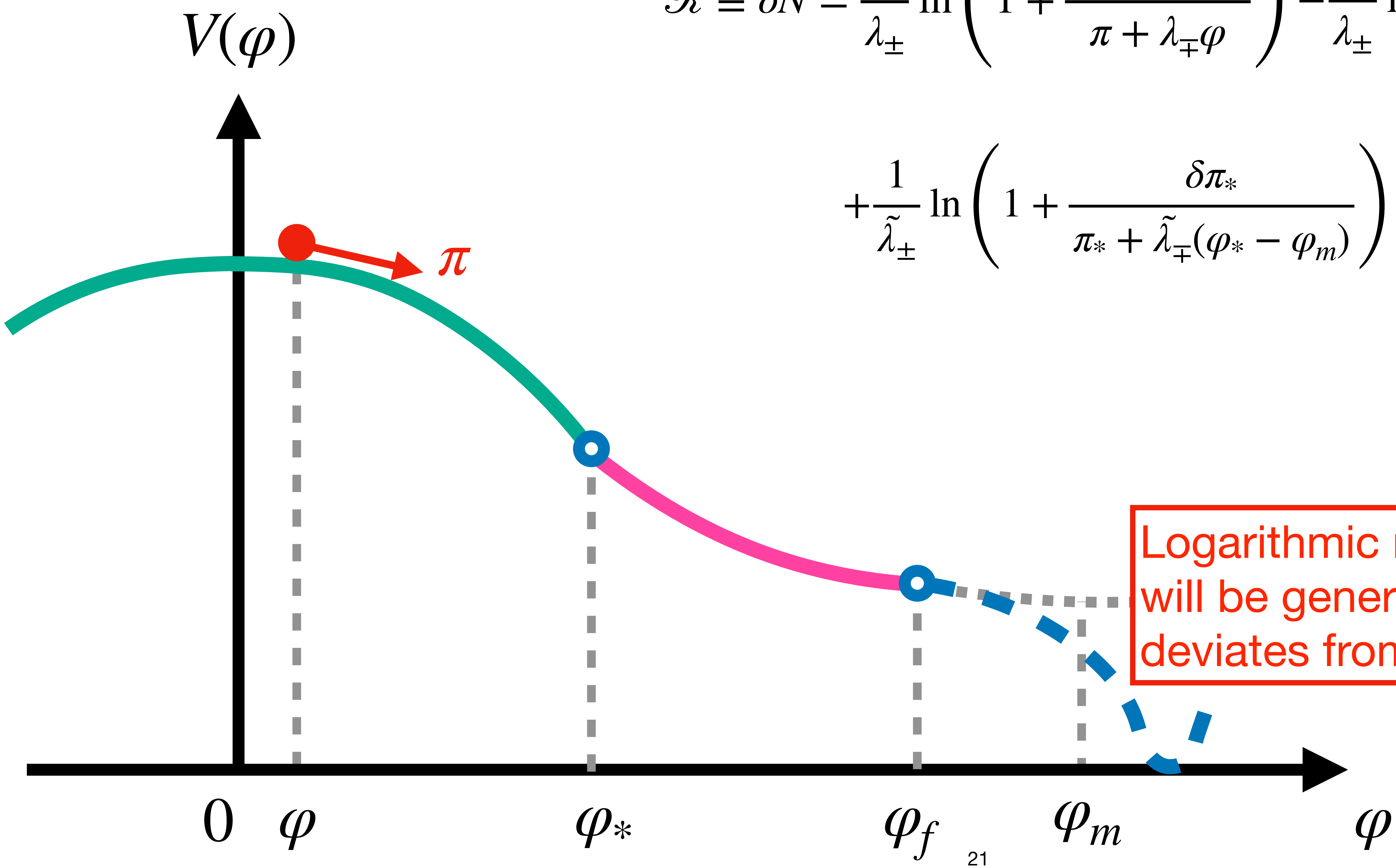
Logarithmic duality of the curvature perturbation

SP and Sasaki, 2211.13932
SP, 2404.06151

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi + \lambda_\mp \delta\varphi}{\pi + \lambda_\mp \varphi}\right) - \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_\mp \varphi_*}\right)$$

$$+ \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_\mp(\varphi_* - \varphi_m)}\right) - \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_f}{\pi_f + \tilde{\lambda}_\mp(\varphi_f - \varphi_m)}\right)$$

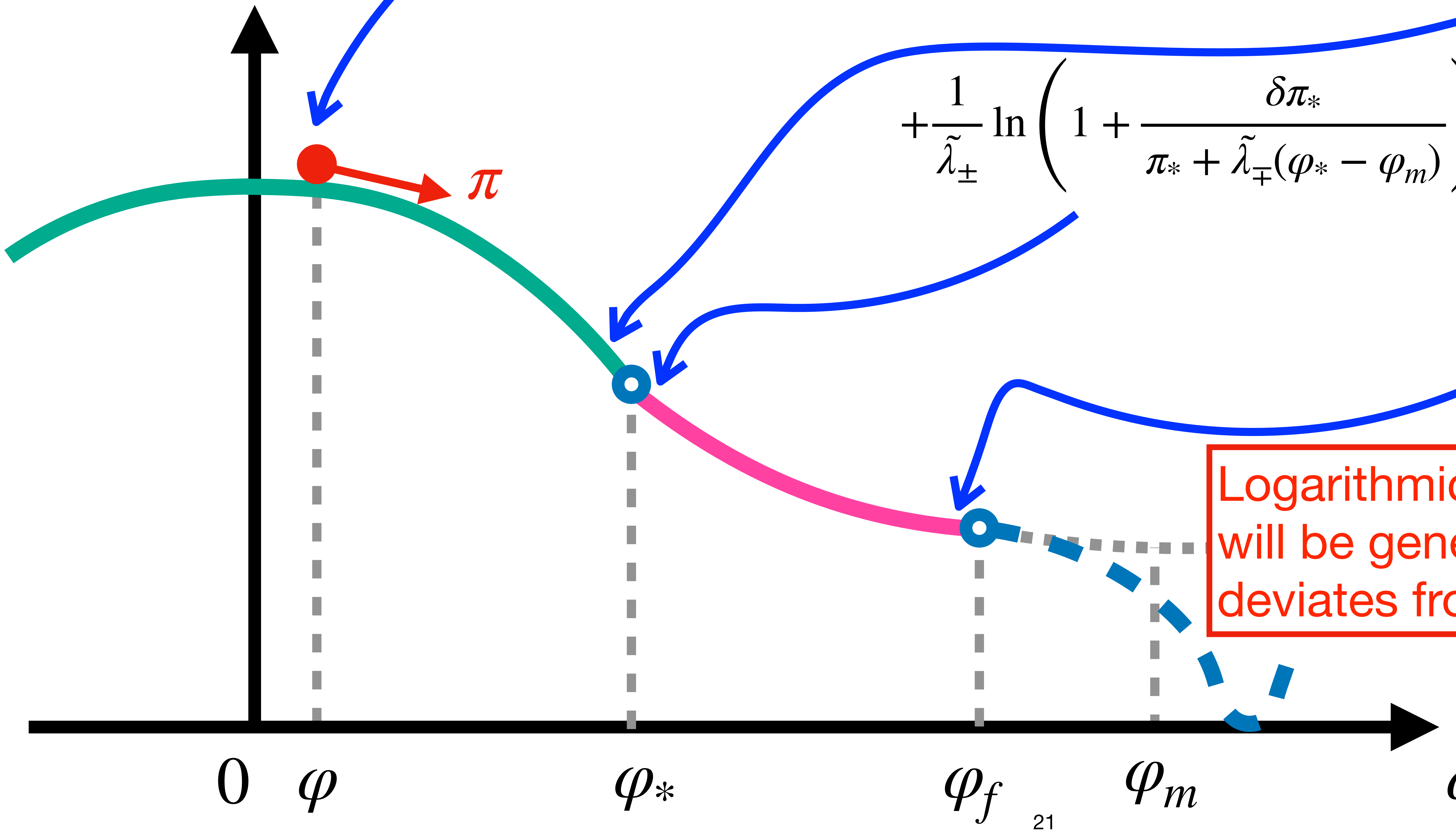


Logarithmic relation (i.e. non-Gaussianity) will be generated when the evolution deviates from attractor.

SP and Sasaki, 2211.13932
SP, 2404.06151

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$
$$+\frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\tilde\lambda_\mp(\varphi_*-\varphi_m)}\right) - \frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_f}{\pi_f+\tilde\lambda_\mp(\varphi_f-\varphi_m)}\right)$$



Logarithmic relation (i.e. non-Gaussianity) will be generated when the evolution deviates from attractor.

SP and Sasaki, 2211.13932
SP, 2404.06151

Logarithmic Duality

SP and Sasaki, 2211.13932
SP, 2404.06151

$$\mathscr{R}(\delta\varphi, \delta\pi)$$

$$\mathscr{R} = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)+\cdots$$

$\lambda_- \ll 1$

$$\left(f_{NL} = -\frac{5}{6}\lambda_-\right)$$

$$\mathscr{R} = -H\frac{\delta\varphi}{\dot\varphi} + \frac{3}{5}f_{\rm NL}\left(-H\frac{\delta\varphi}{\dot\varphi}\right)^2$$

Slow-roll inflation
Stewart and Sasaki, 1995
Lyth and Roquigez, 2005

$\lambda_- = -1/\mu$

$$\mathscr{R} = -\mu\ln\left(1-\frac{\mathscr{R}_g}{\mu}\right)$$

Constant-roll
Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990
Inui, Motohashi, SP, et al, 2409.13500

$\lambda_- = 0$

$$\mathscr{R} = -\frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll
Namjoo, Firouzjahi, Sasaki, 1210.3692
Cai, Chen, et al 1712.09998
Biagetti et al 1804.07124
Passaglia et al 1812.08243

$\lambda_- = 3/2$

$$\mathscr{R} = \frac{2}{3}\ln(1+\delta)$$

Curvaton scenario,
SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728
Hooper et al. 2308.00756

$$\mathscr{R} = -\frac{1}{\lambda}\ln\left(f(\mathscr{R}_{\rm G})\right)$$

Extensions,
Kawaguchi et al, 2305.18140
SP and Yokoyama, in prep

Probability Distribution Function

For the simplest single-logarithm case:

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+\delta\varphi}{\pi + \lambda_+\varphi}\right)$$

$$\Downarrow \quad P(\mathscr{R})d\mathscr{R} = \boxed{P(\delta\varphi)}\, d\delta\varphi$$

→ Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$P(\mathscr{R}) = \frac{e^{\lambda_-\mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma_{\delta\varphi}^2}\left(e^{\lambda_-\mathscr{R}} - 1\right)^2\right]$$

SP and Sasaki, 2211.13932

Probability Distribution Function

For the simplest single-logarithm case: $\mathscr{R} \equiv \delta N = \dfrac{1}{\lambda_-} \ln\left(1 + \dfrac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right)$

$$\Downarrow \quad P(\mathscr{R})d\mathscr{R} = P(\delta\varphi)d\delta\varphi$$

$P(\delta\varphi)$ → Gaussian PDF with variance $\sigma^2_{\delta\varphi}$

$$P(\mathscr{R}) = \frac{e^{\lambda_- \mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma^2_{\delta\varphi}}\left(e^{\lambda_- \mathscr{R}} - 1\right)^2\right]$$

$\lambda_- < 0$ → $P(\mathscr{R}) \sim e^{\lambda_- \mathscr{R}}$

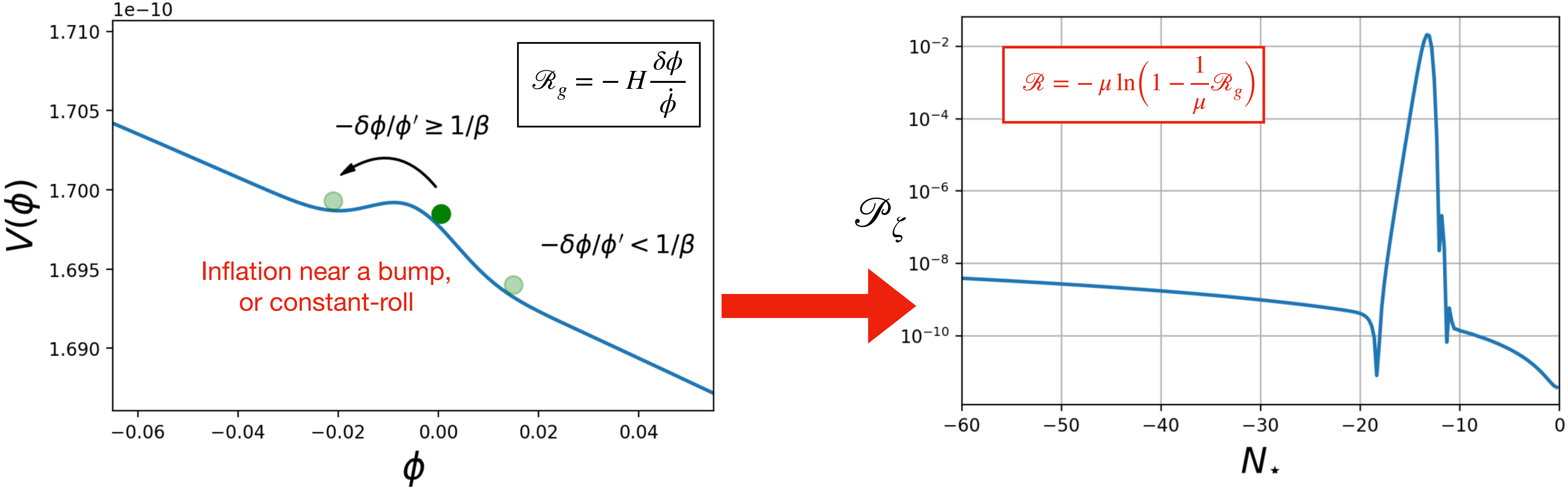
exponential tail

$\lambda_- > 0$ → $P(\mathscr{R}) \sim \exp\left(-c^2 e^{2\lambda_- \mathscr{R}}\right)$

Gumbel-distribution-like tail

SP and Sasaki, 2211.13932

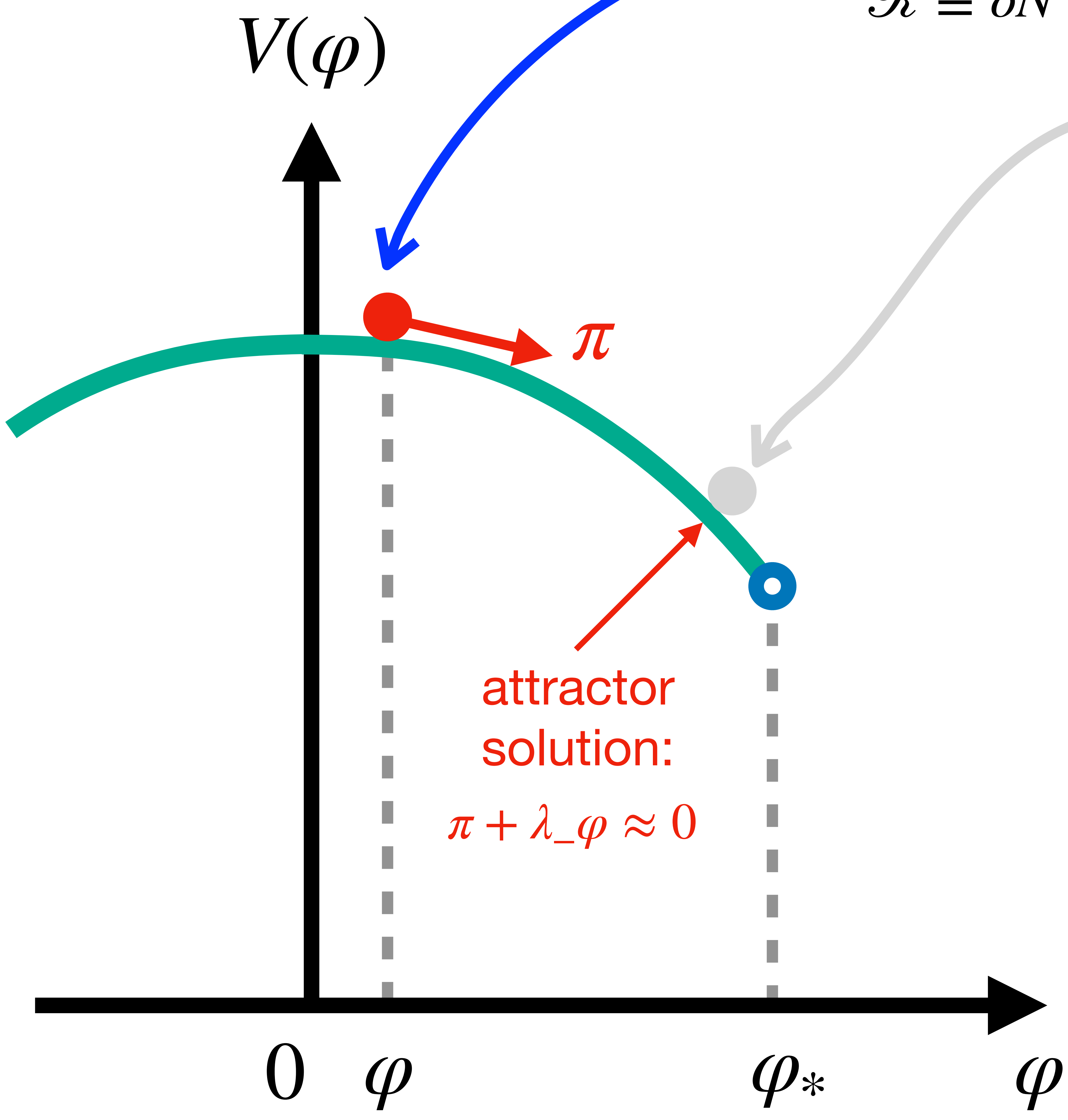
Case1: Constant-roll

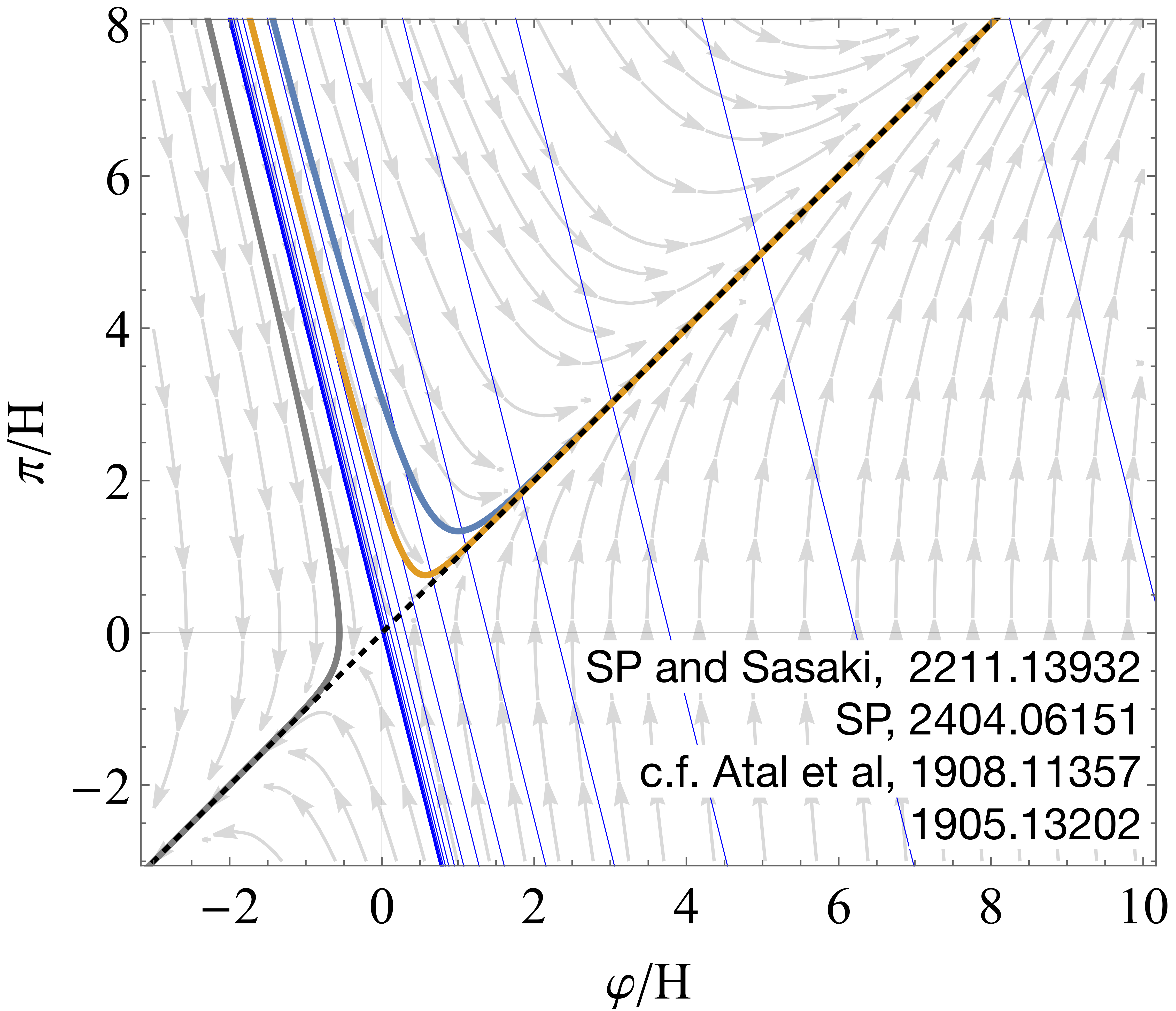


Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990

Case1: Constant-roll

$$\mathcal{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+\delta\varphi}{\pi + \lambda_+\varphi}\right) - \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_+\varphi_*}\right)$$



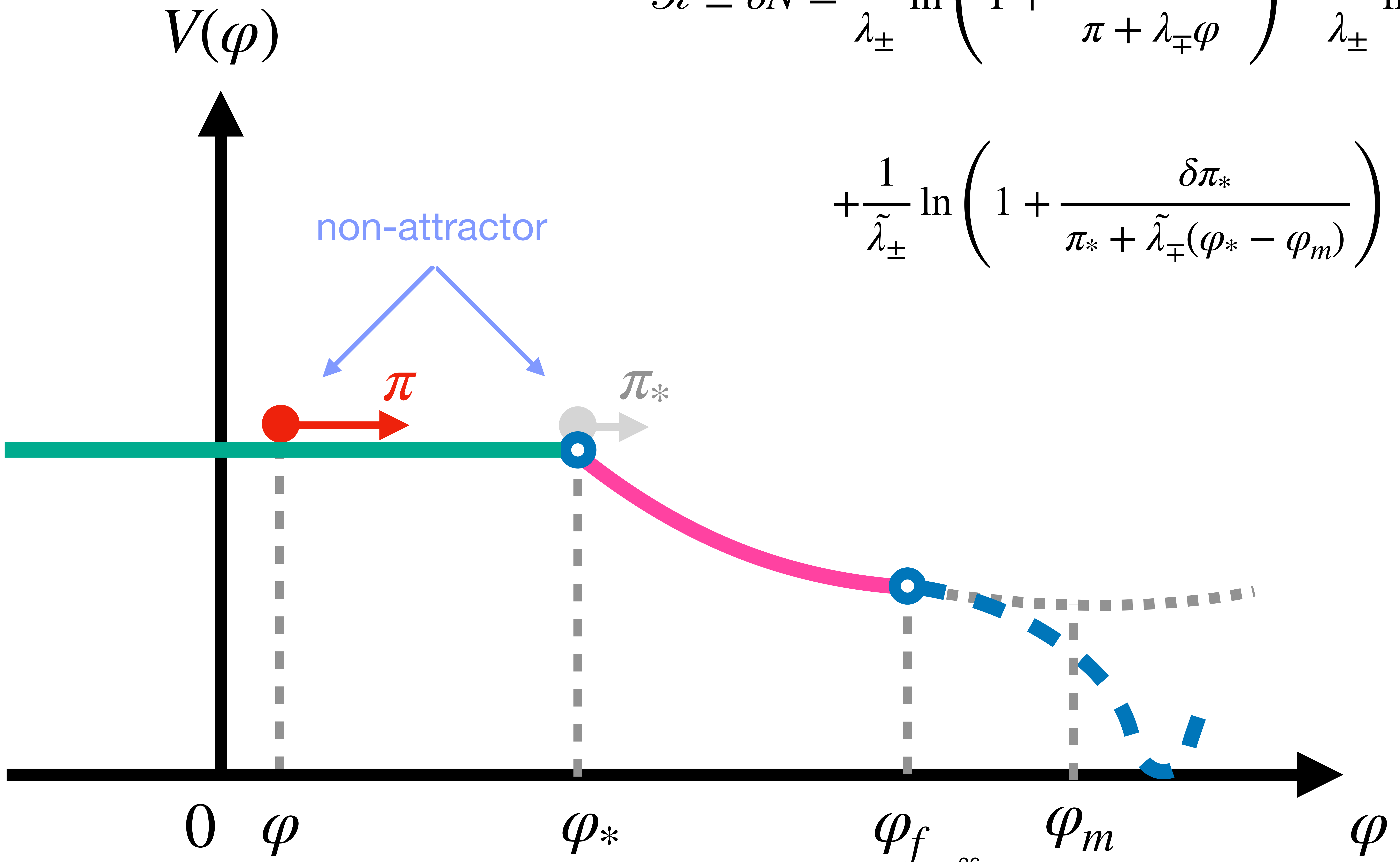


Case 2: USR

$$(\lambda_- = 0, \quad \lambda_+ = 3)$$
$$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$$

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi + \lambda_\mp \delta\varphi}{\pi + \lambda_\mp \varphi}\right) - \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_\pm \varphi_*}\right)$$

$$+ \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_\mp(\varphi_* - \varphi_m)}\right) - \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_f}{\pi_f + \tilde{\lambda}_\pm(\varphi_f - \varphi_m)}\right)$$

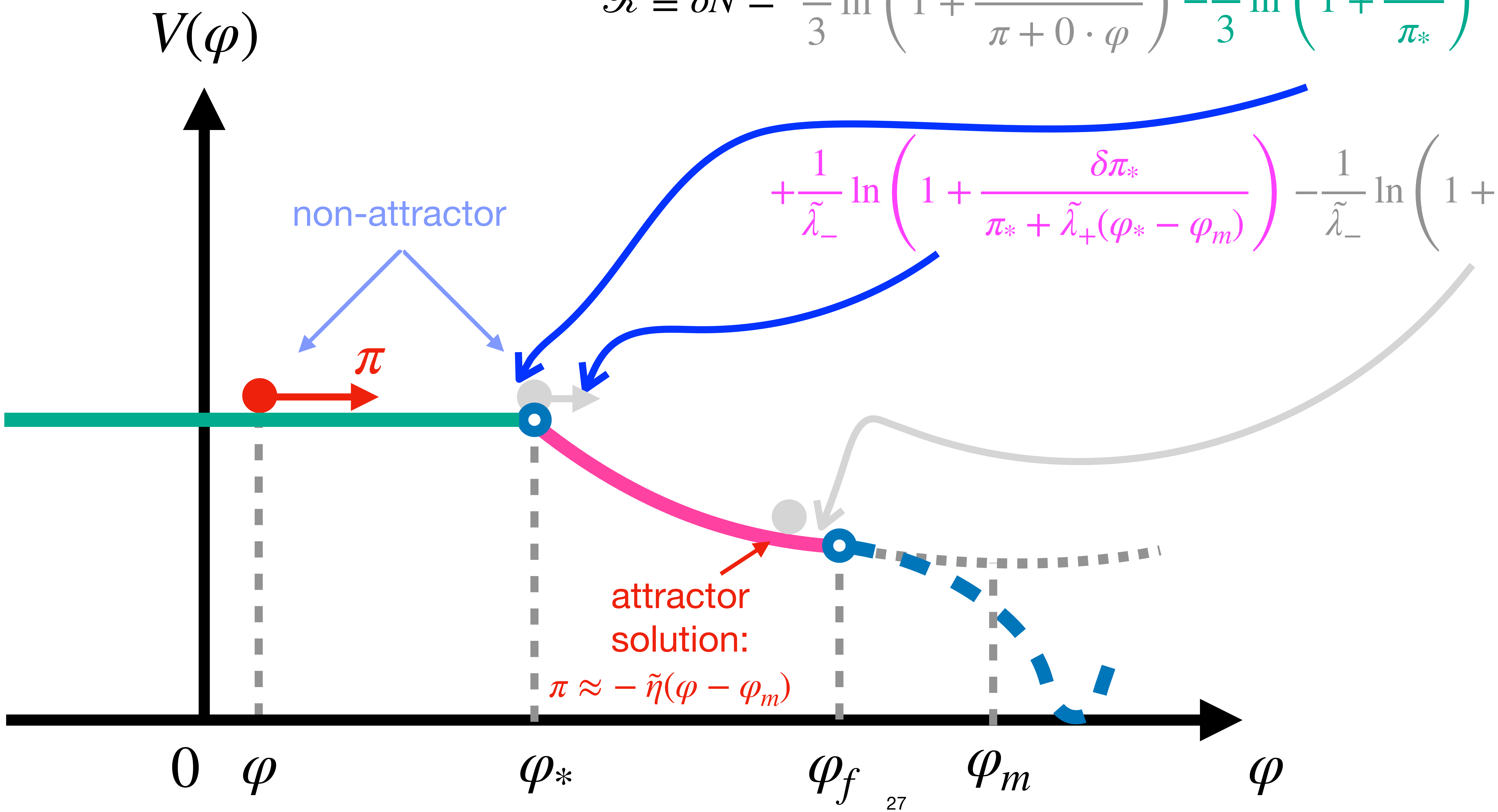


Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$

$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathscr{R} \equiv \delta N = \frac{1}{3}\ln\left(1 + \frac{0 + 0\cdot\delta\varphi}{\pi + 0\cdot\varphi}\right) - \frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\lambda}_-}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_+(\varphi_* - \varphi_m)}\right) - \frac{1}{\tilde{\lambda}_-}\ln\left(1 + \frac{\delta\pi_f}{\pi_f + \tilde{\lambda}_+(\varphi_f - \varphi_m)}\right)$$



Case 2: USR

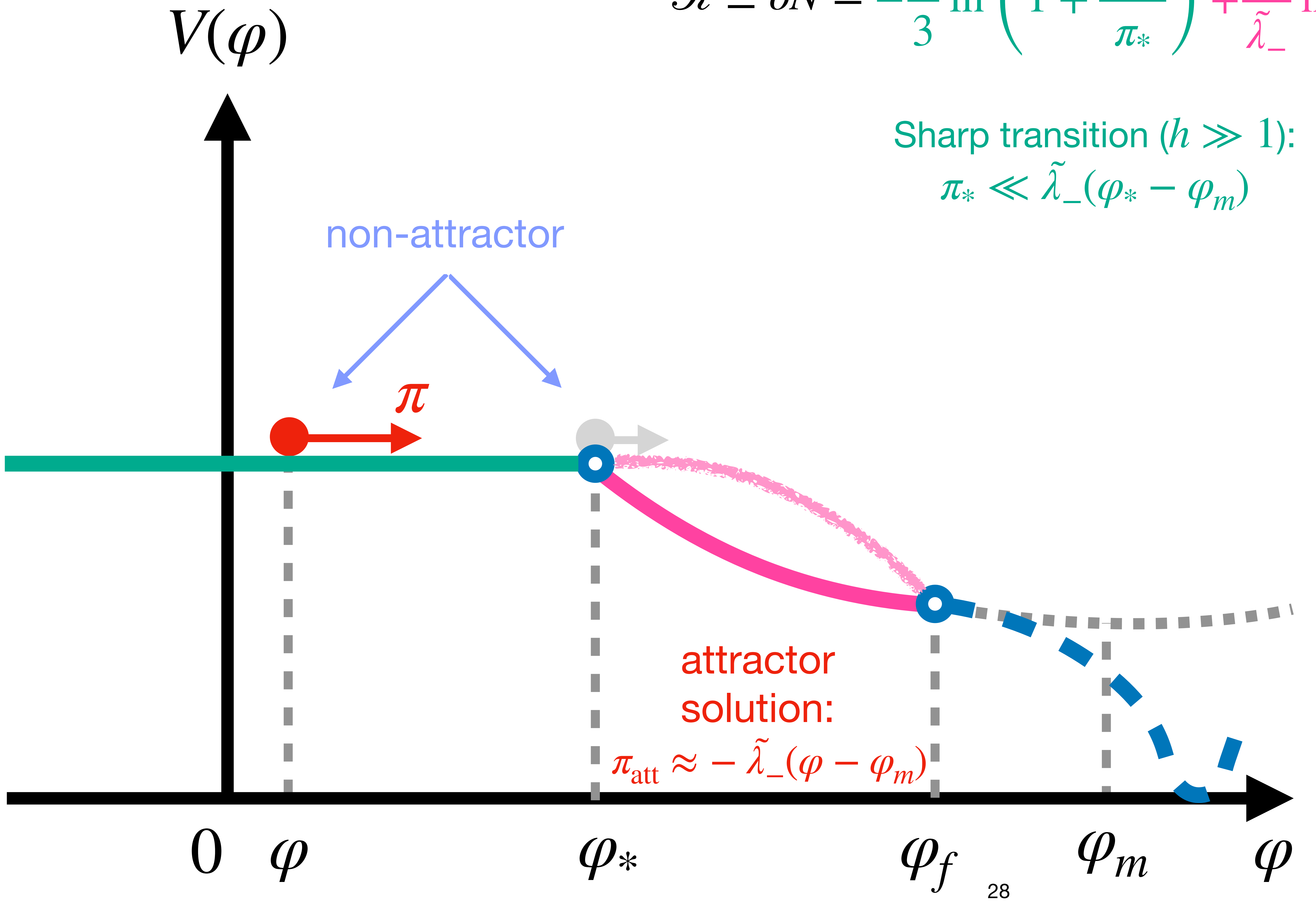
$(\lambda_- = 0, \quad \lambda_+ = 3)$

$(\tilde{\lambda}_- \approx \tilde{\eta}, \quad \tilde{\lambda}_+ \approx 3 - \tilde{\eta})$

$$\mathscr{R} \equiv \delta N = -\frac{1}{3} \ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\lambda}_-} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_+(\varphi_* - \varphi_m)}\right)$$

Sharp transition ($h \gg 1$):
$\pi_* \ll \tilde{\lambda}_-(\varphi_* - \varphi_m)$

Smooth transition ($h \ll 1$)
$\pi_* \gg \tilde{\lambda}_-(\varphi_* - \varphi_m)$



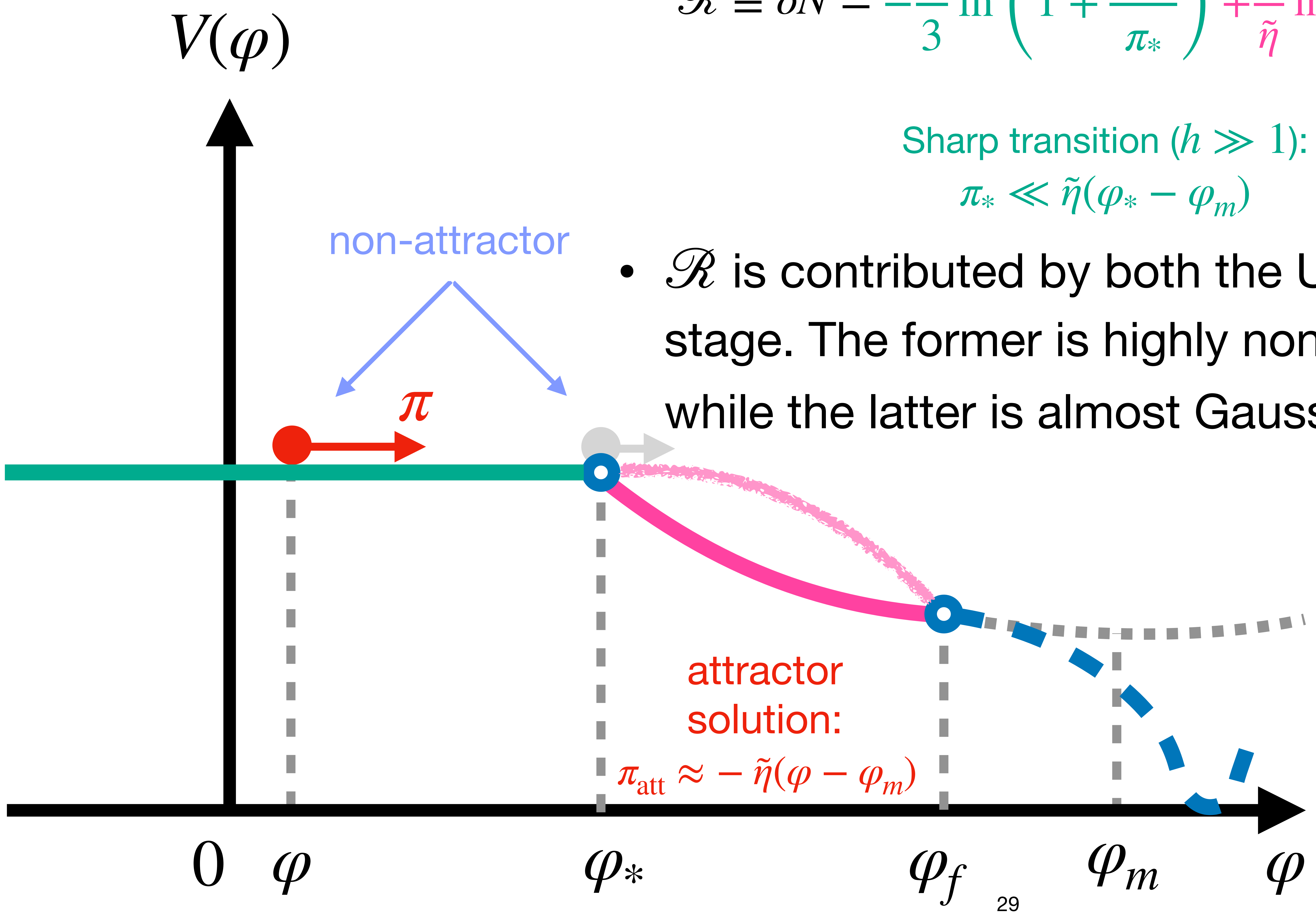
Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- \approx \tilde{\eta}, \quad \tilde{\lambda}_+ \approx 3 - \tilde{\eta})$

$$\mathcal{R} \equiv \delta N = -\frac{1}{3} \ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

Sharp transition ($h \gg 1$): $\pi_* \ll \tilde{\eta}(\varphi_* - \varphi_m)$

Smooth transition ($h \ll 1$) $\pi_* \gg \tilde{\eta}(\varphi_* - \varphi_m)$



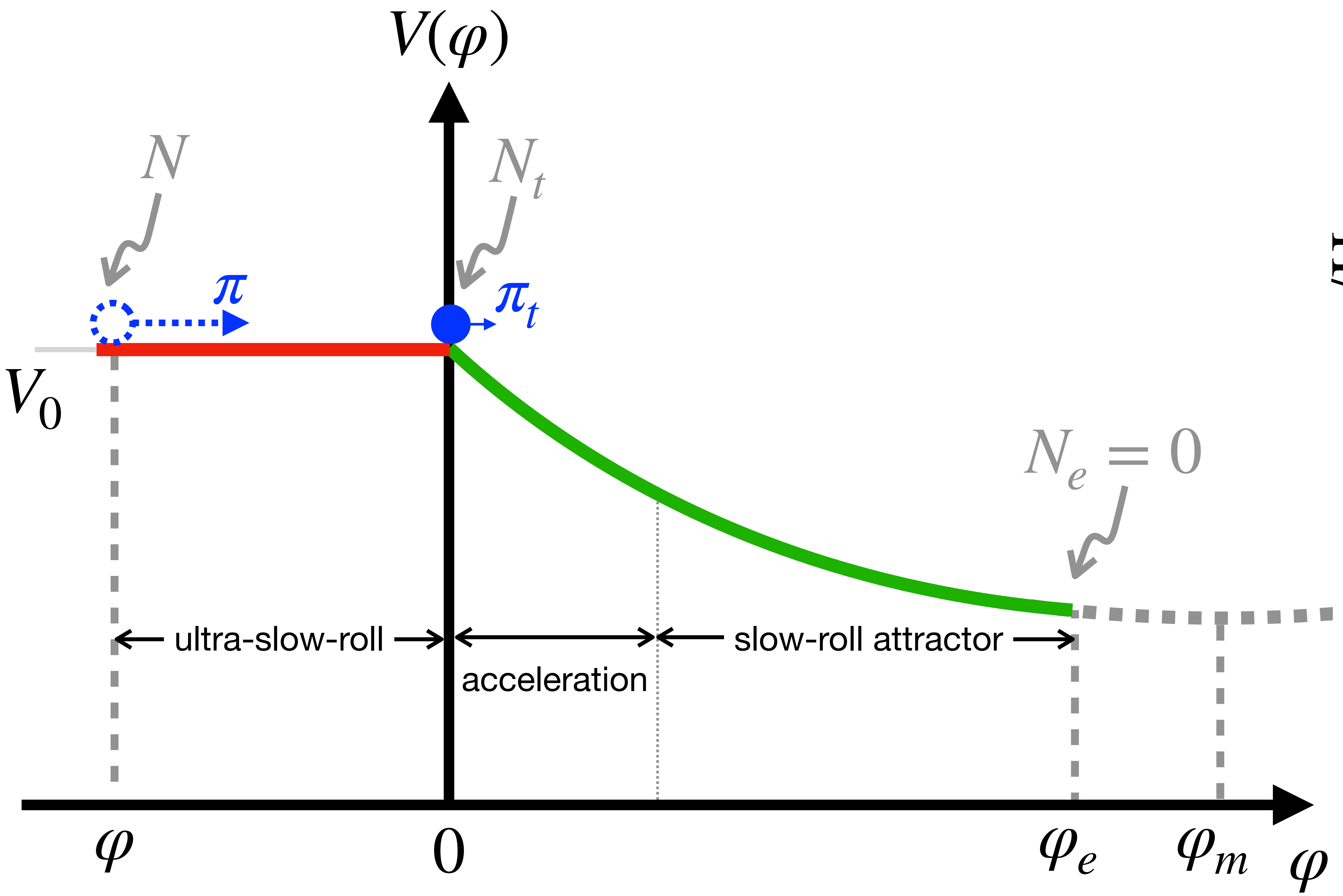
- $\mathcal{R}$ is contributed by both the USR stage and the later slow-roll stage. The former is highly non-Gaussian (i.e. exp tail, $f_{\rm NL} = 5/2$), while the latter is almost Gaussian ($f_{\rm NL} = -5\tilde{\eta}/6$).

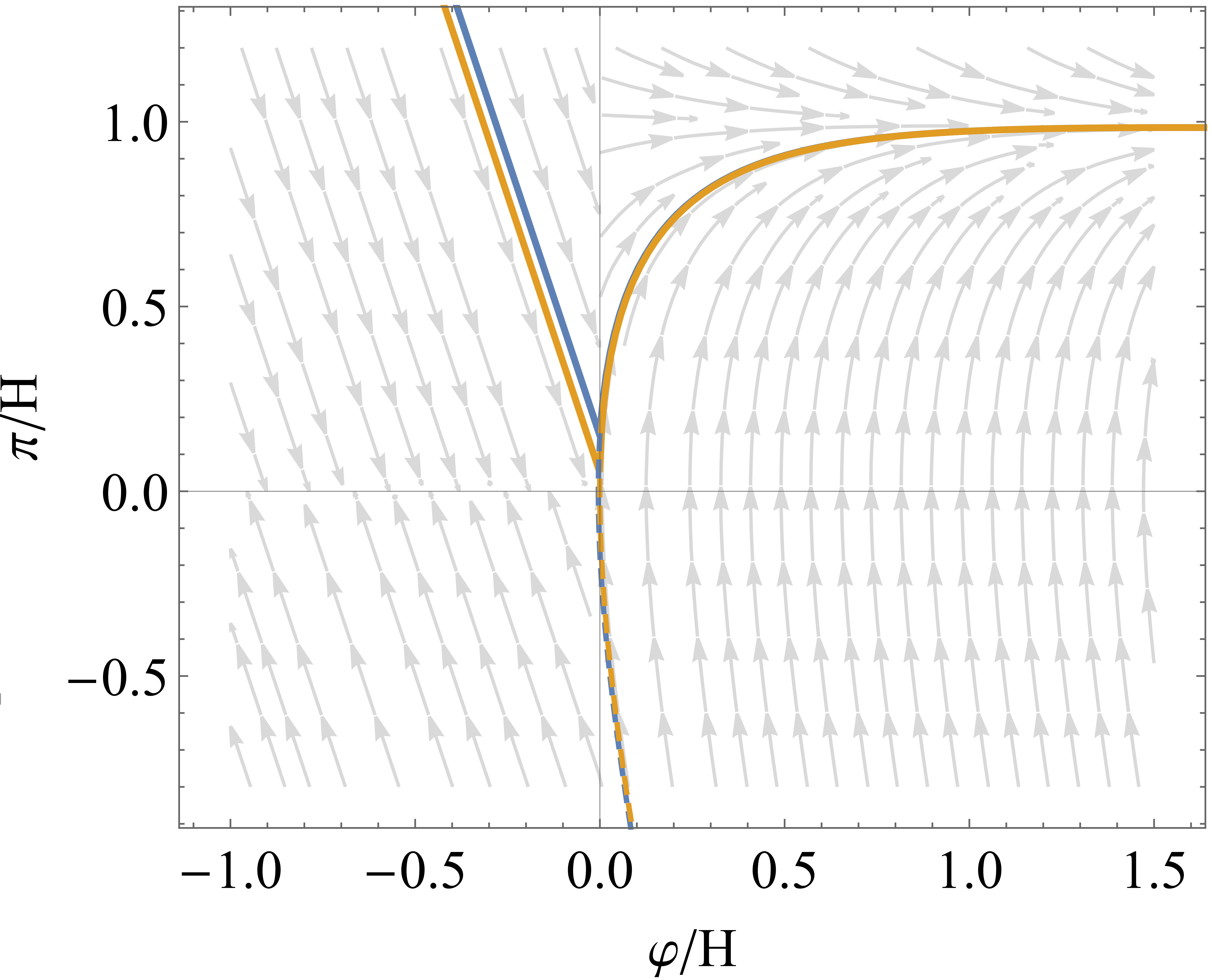
- c.f. The h defined in 1712.09998

$$h \equiv -6\sqrt{\frac{\epsilon_V}{\epsilon_*}} = 6\left|\frac{\lambda_-(\varphi_* - \varphi_m)}{\pi_*}\right|$$

- When $h \sim \mathcal{O}(1)$, both of the logarithms are of the same order.

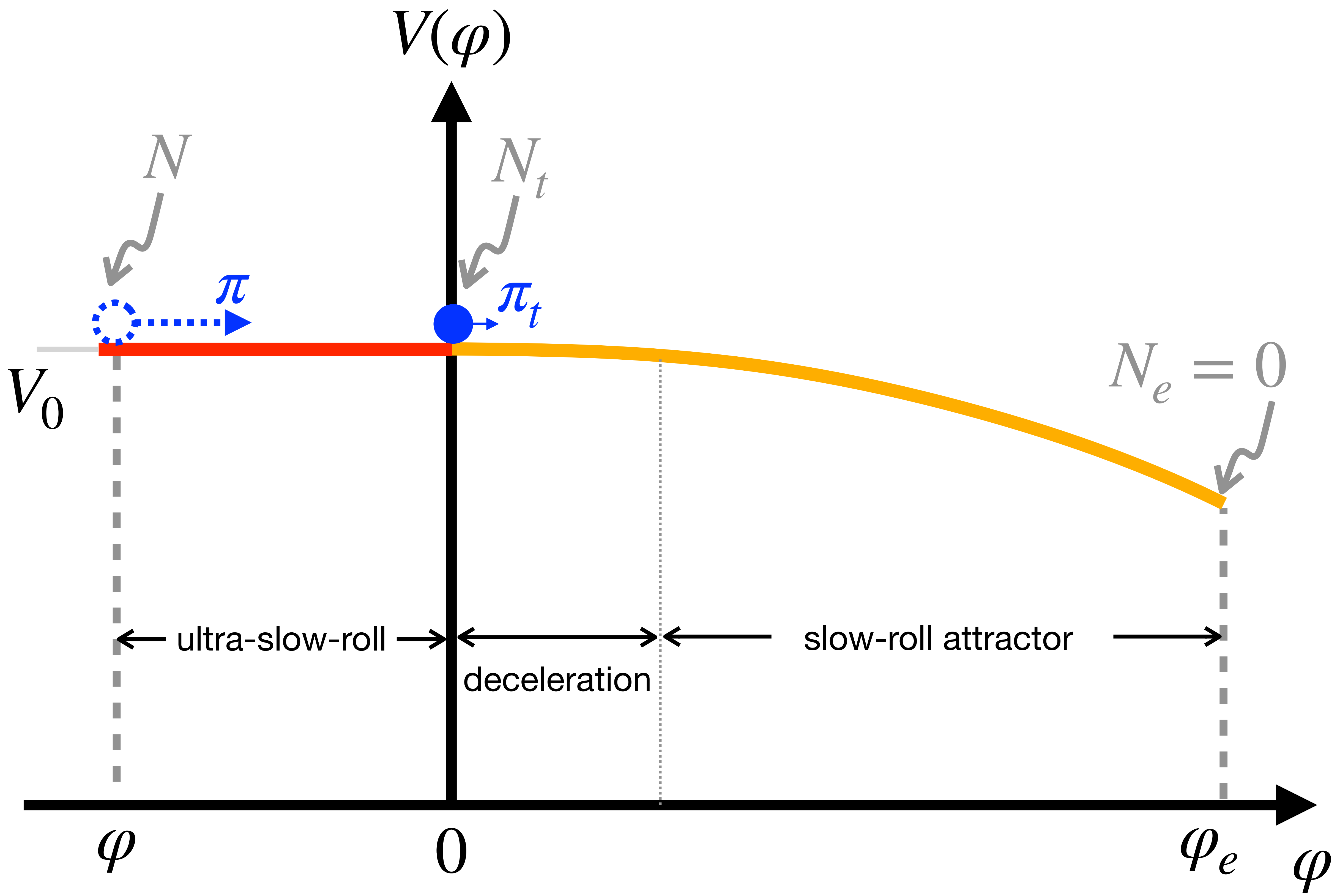
USR: Sharp end

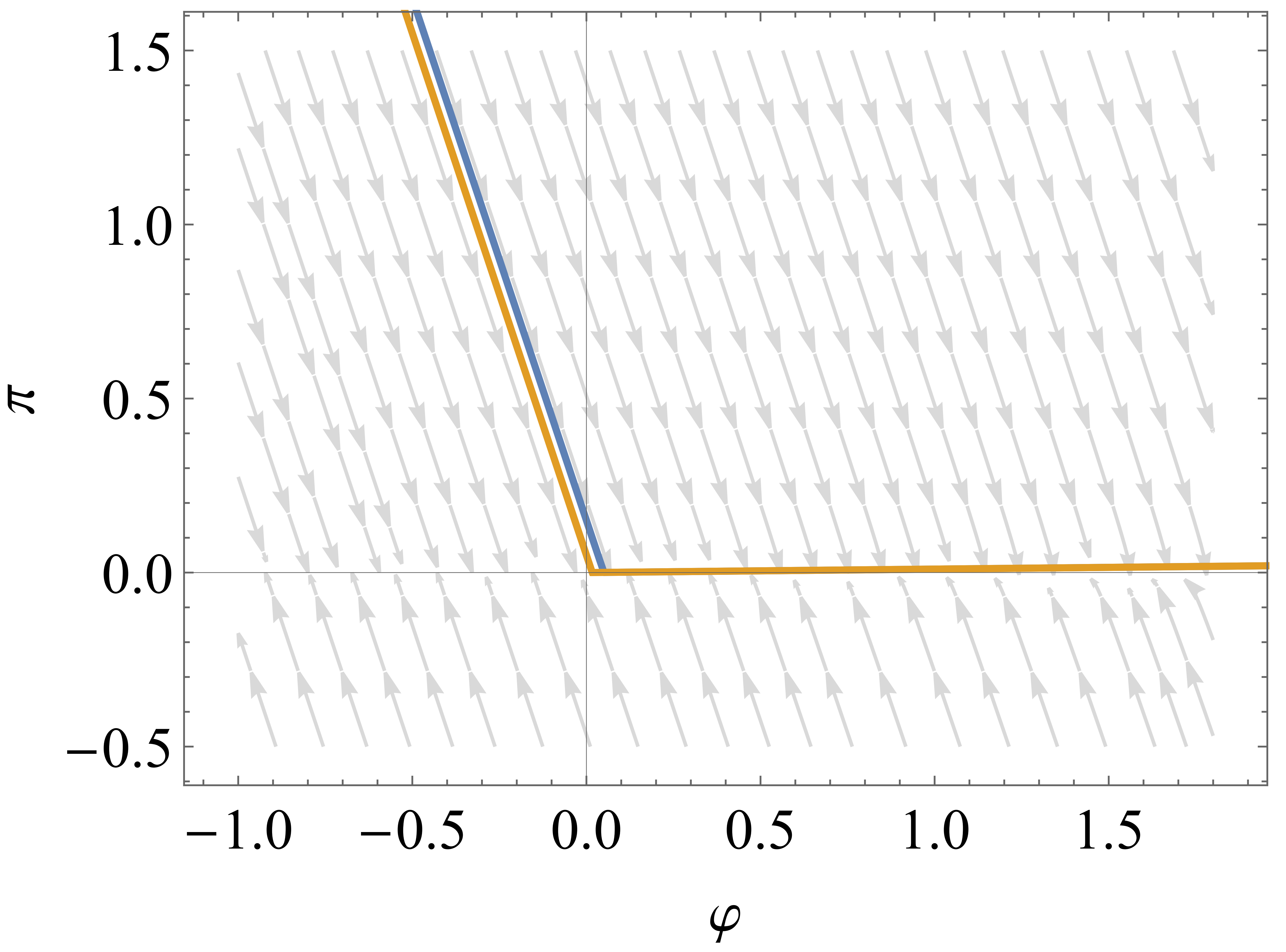




SP and Sasaki, 2211.13932
SP, 2404.06151
c.f. Cai et al, 1712.09998

USR: Smooth end



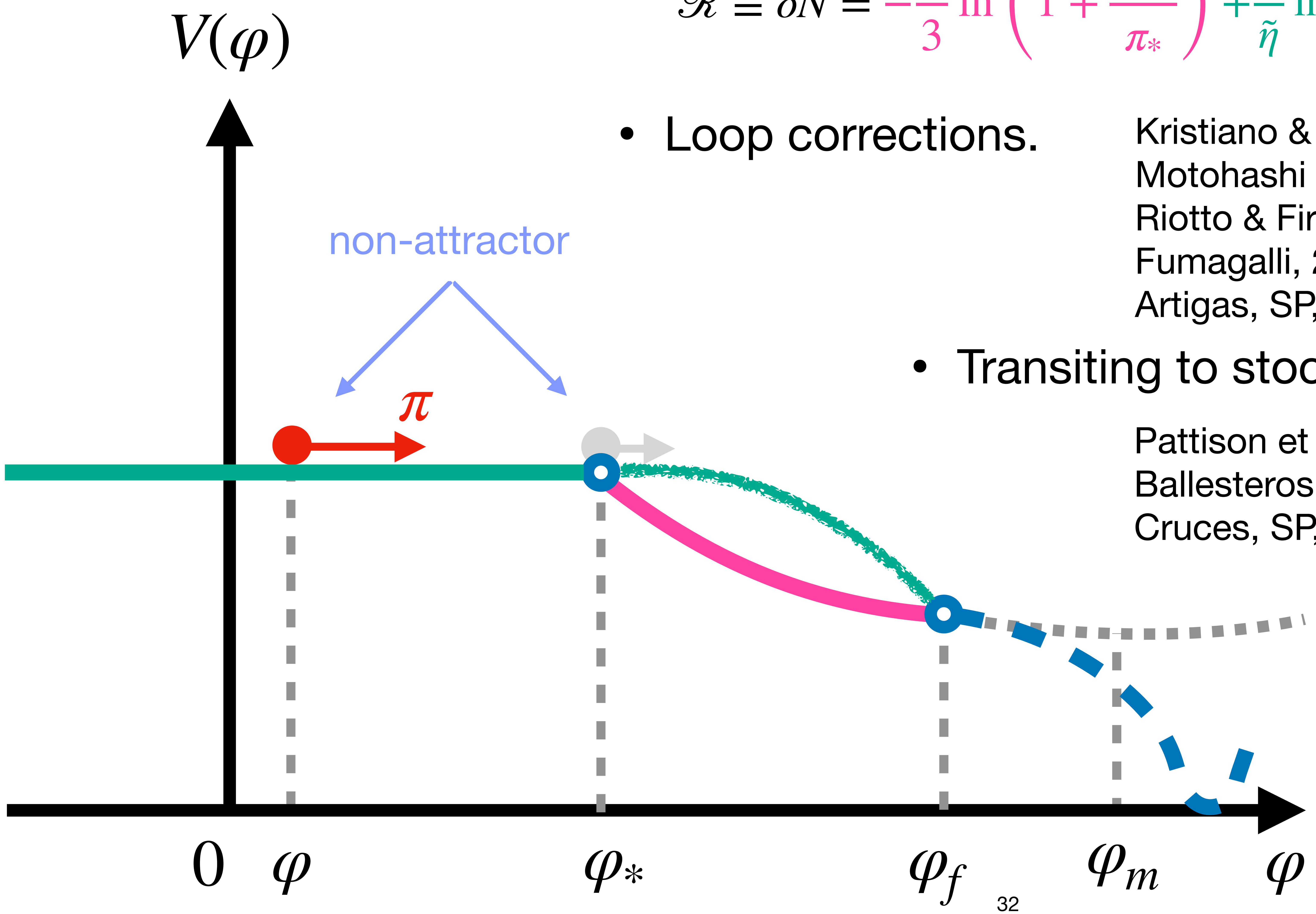


SP and Sasaki, 2211.13932
SP, 2404.06151
c.f. Cai et al, 1712.09998

USR

$$(\lambda_- = 0, \quad \lambda_+ = 3)$$
$$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$$

$$\mathscr{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$



- Loop corrections.

Kristiano & Yokoyama, 2211.03395, 2405.12145
Motohashi & Tada, 2303.16035
Riotto & Firouzjahi, 2304.07801
Fumagalli, 2305.19263
Artigas, SP, Tanaka, Urakawa, to appear

Laura's talk

- Transiting to stochastic approach

Pattison et al., 2101.05741
Ballesteros et al 2406.02417
Cruces, SP, Sasaki, in preparation.

Guillermo's talk

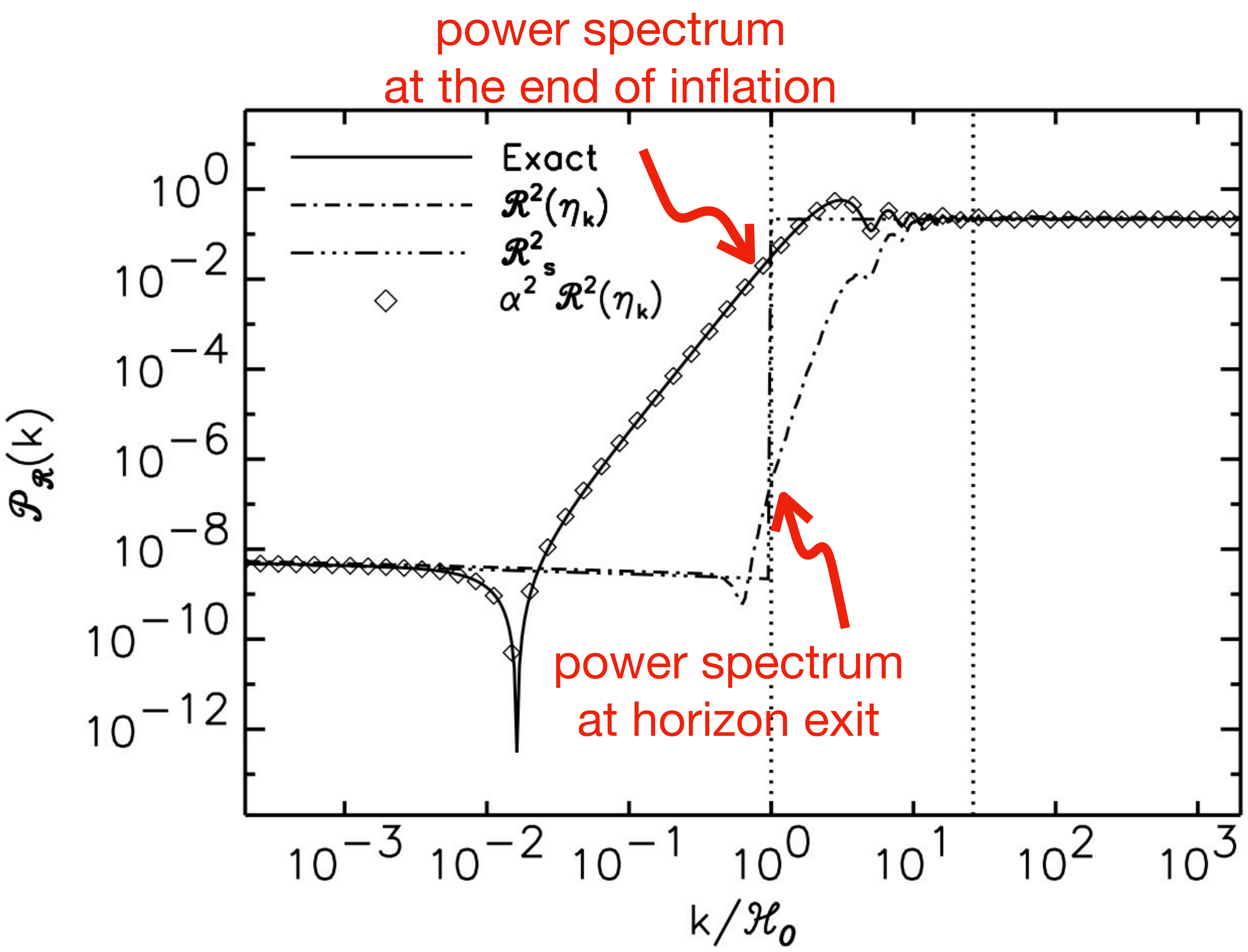
- Sharp transition will make the separate universe approach (thus δN formalism) invalid transiently.

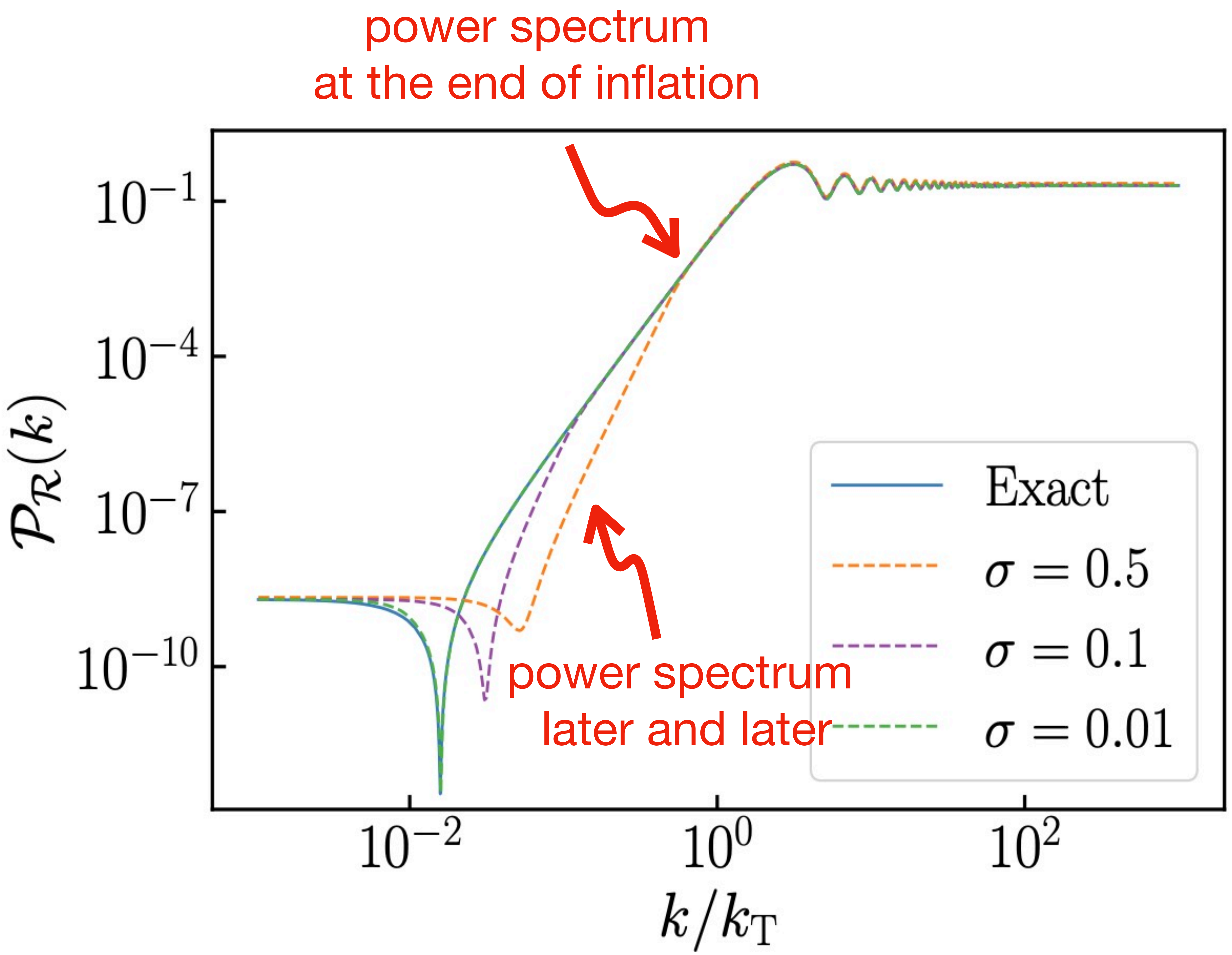
Domenech et al., 2309.05750
Jackson et al., 2311.03281
Artigas, SP, Tanaka, 2408.09964

CONTENT

- Introduction: δN formalism
- Application: USR and general single-field
- Recent topics: gradient expansion and bubbles
- Conclusion

Ex1: Separate Universe

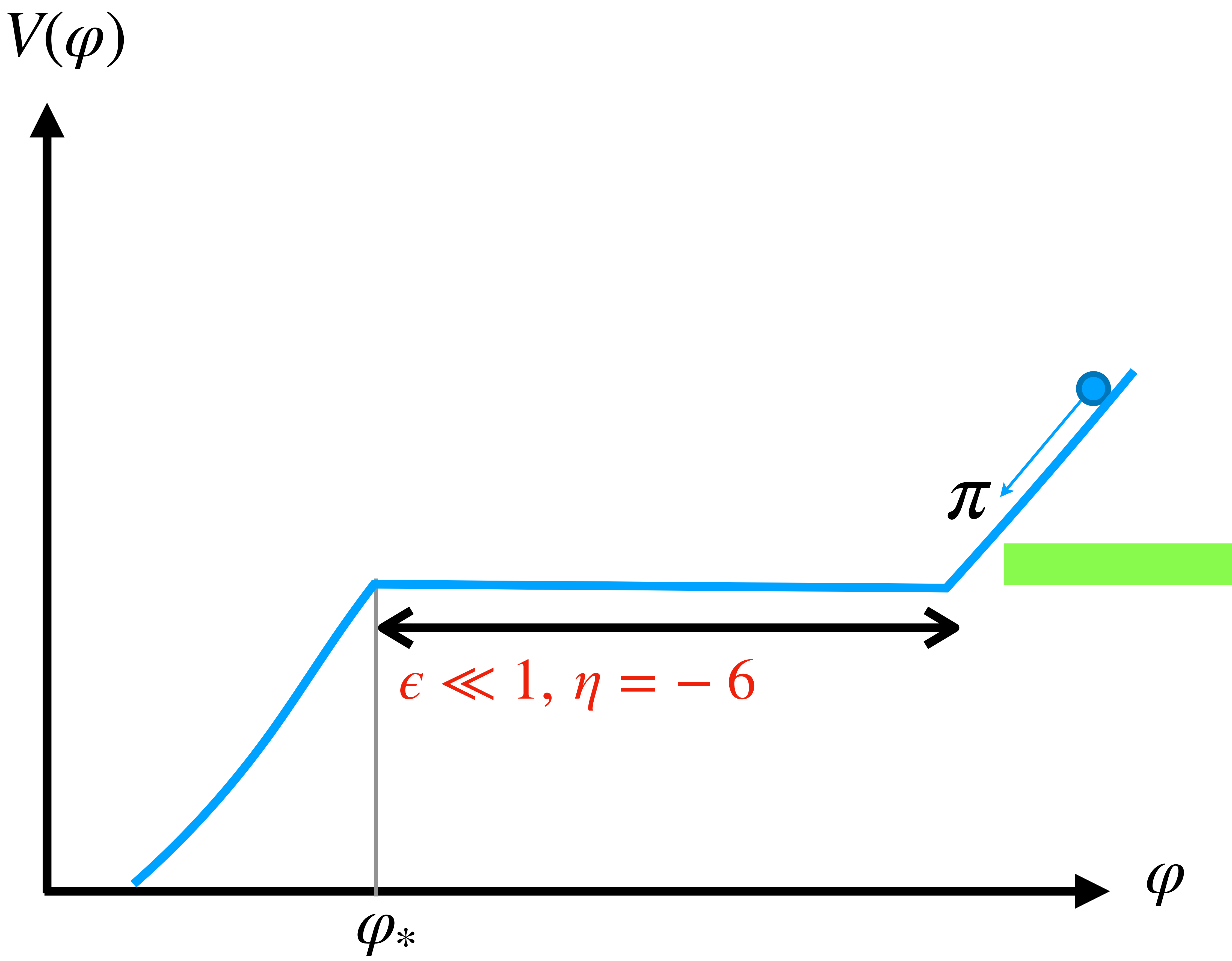


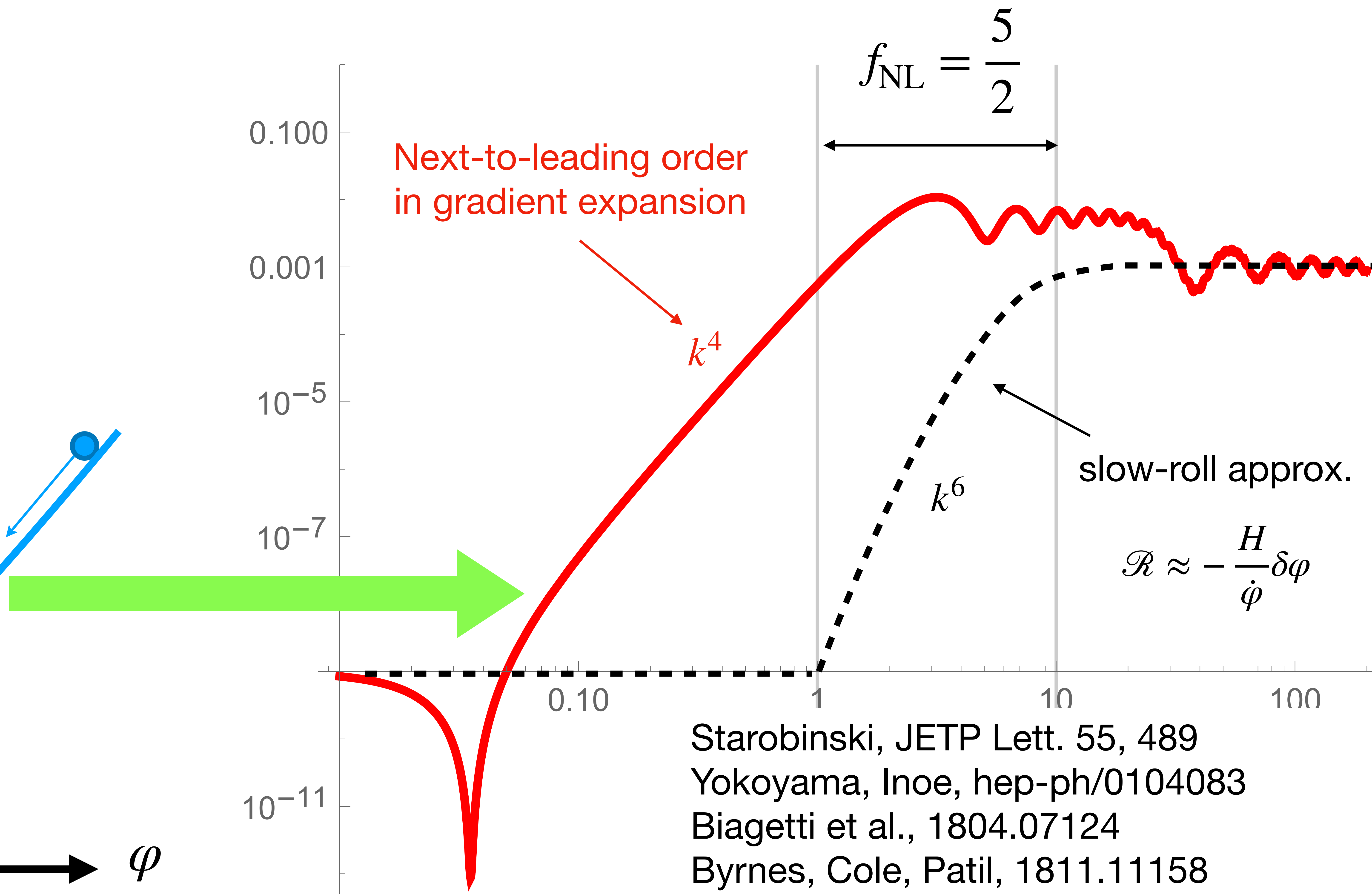


Leach, Sasaki, Wands, Liddle, astro-ph/0101406
SP and Jianing Wang, 2209.14183

Domenech et al., 2309.05750
Jackson et al., 2311.03281

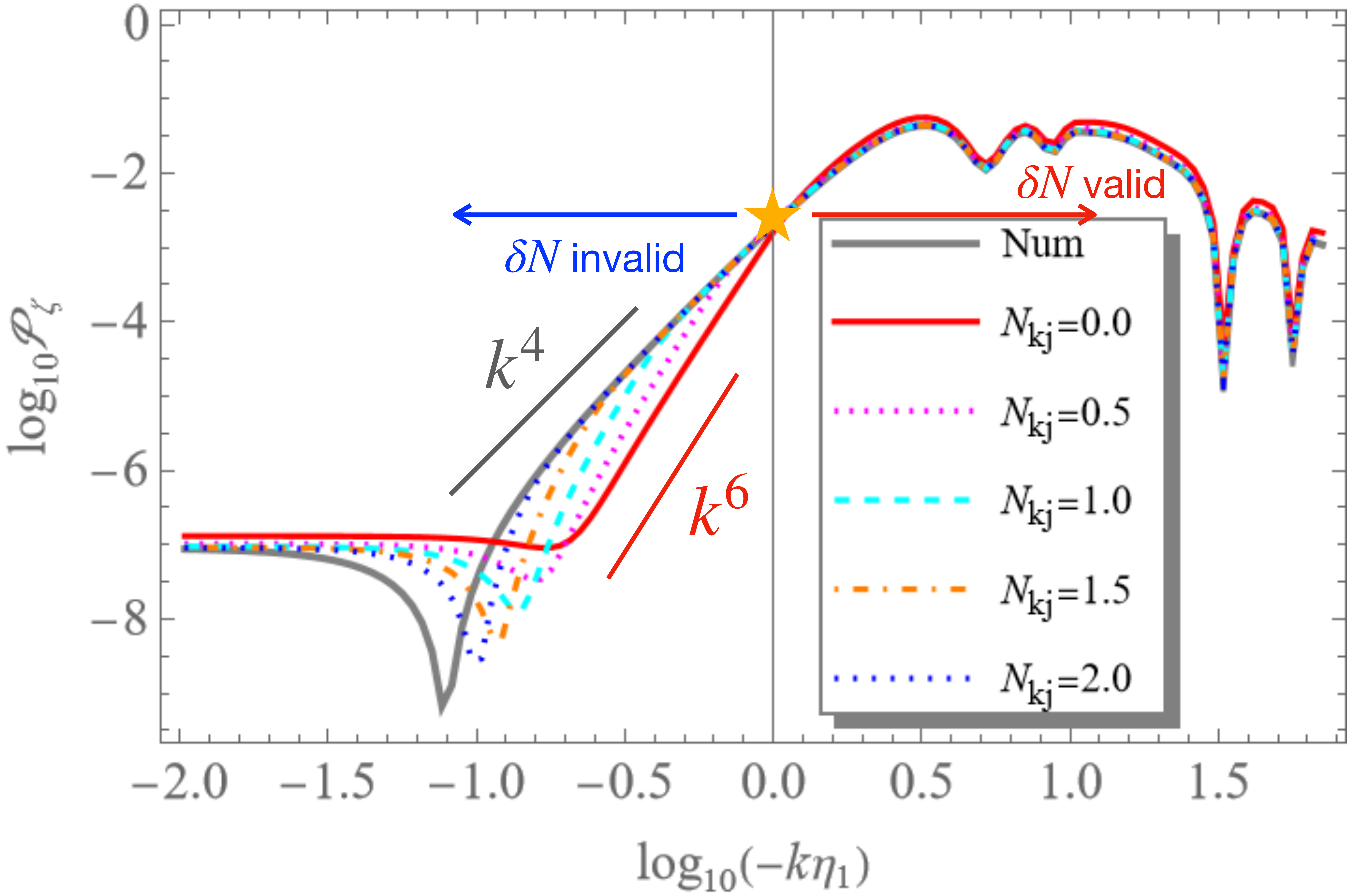
Ultra-slow-roll inflation

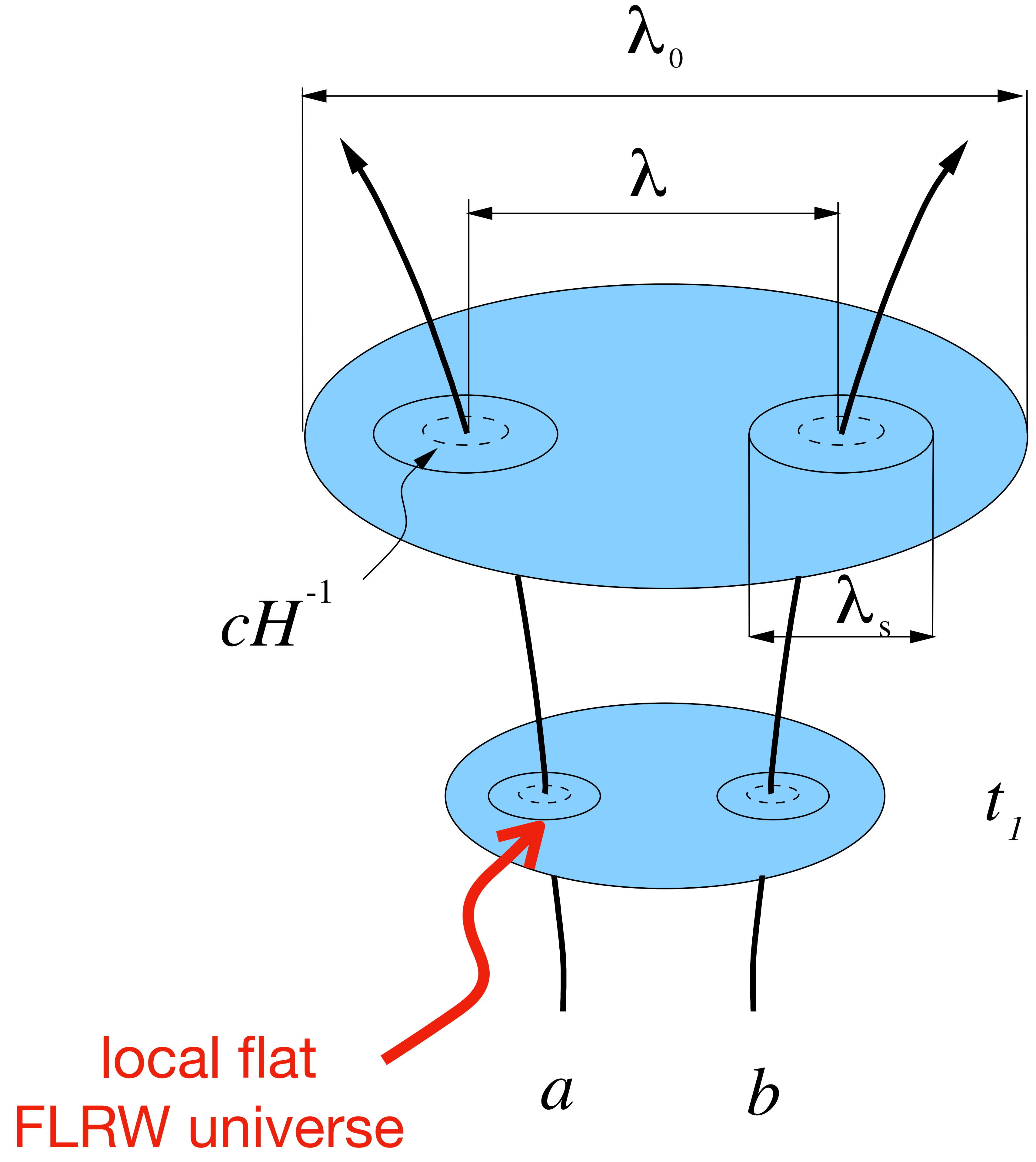




Starobinski, JETP Lett. 55, 489
Yokoyama, Inoe, hep-ph/0104083
Biagetti et al., 1804.07124
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP, Jianing Wang, 2209.14183
Domenech, Vargas, Vargas, 2309.05750

Separate Universe

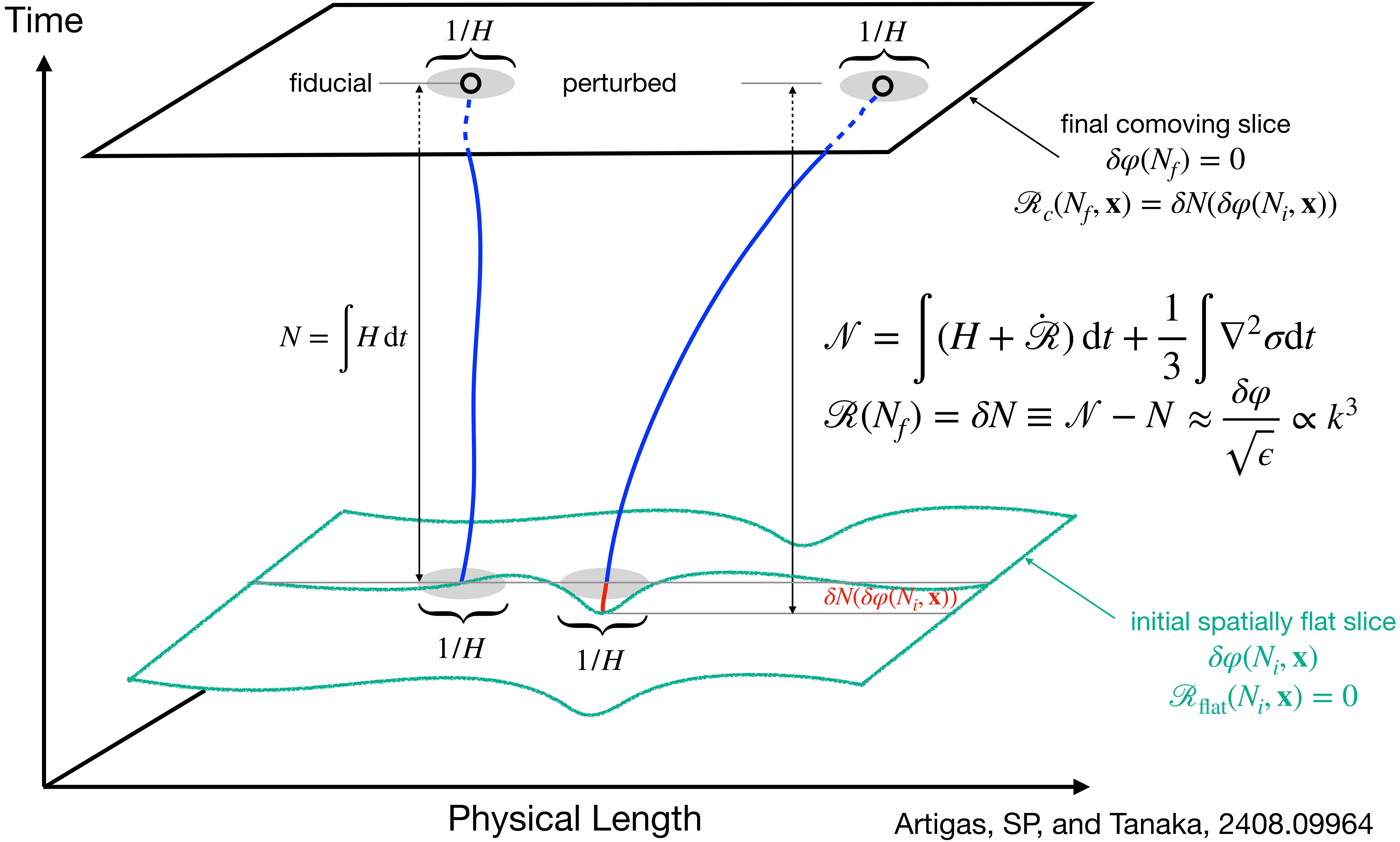




$$\mathscr{R}(N_e) = \delta N(\delta\varphi(N_i, \mathbf{x}))$$

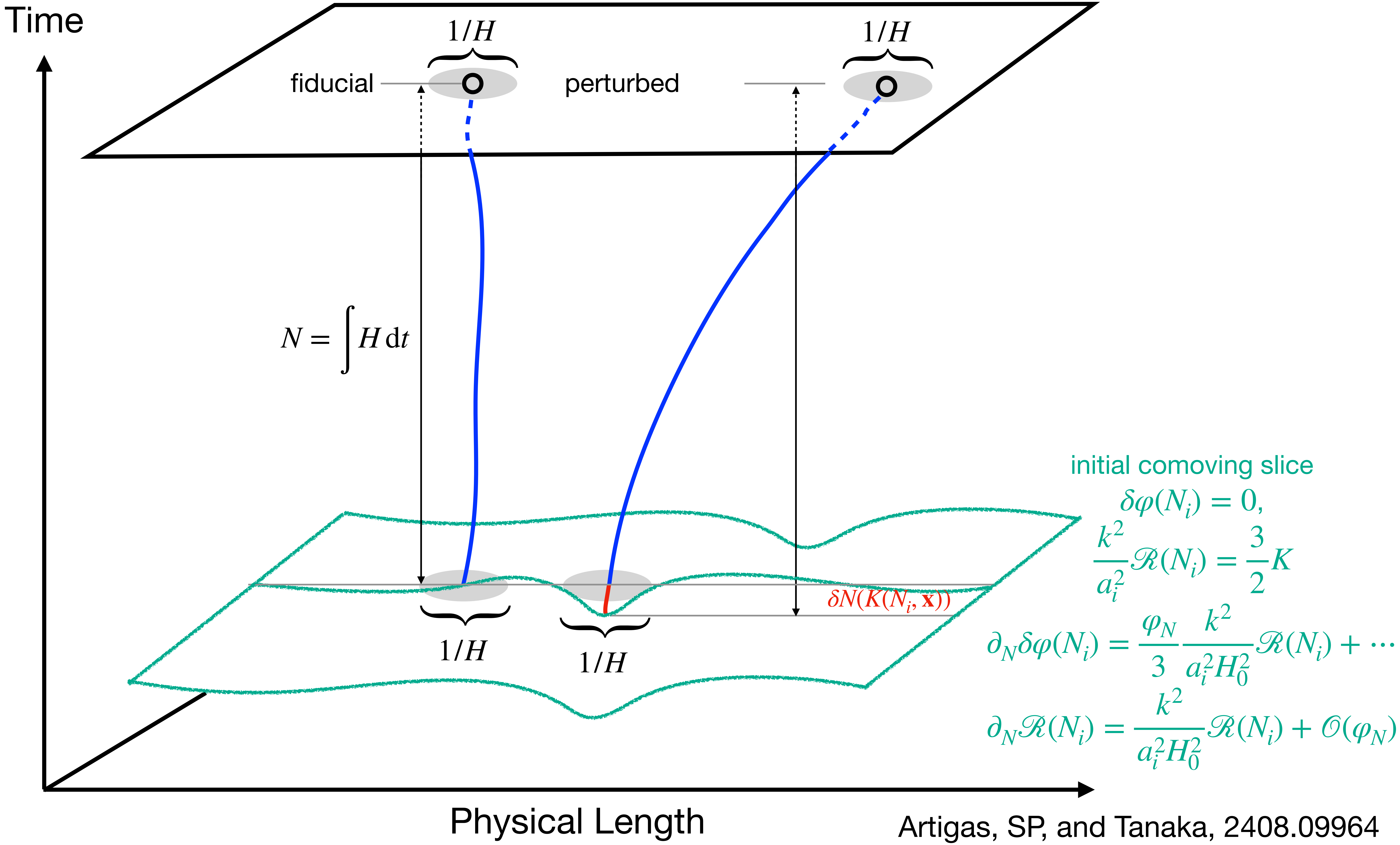
c.f.
Domenech et al., 2309.05750
Jackson et al., 2311.03281

δN formalism



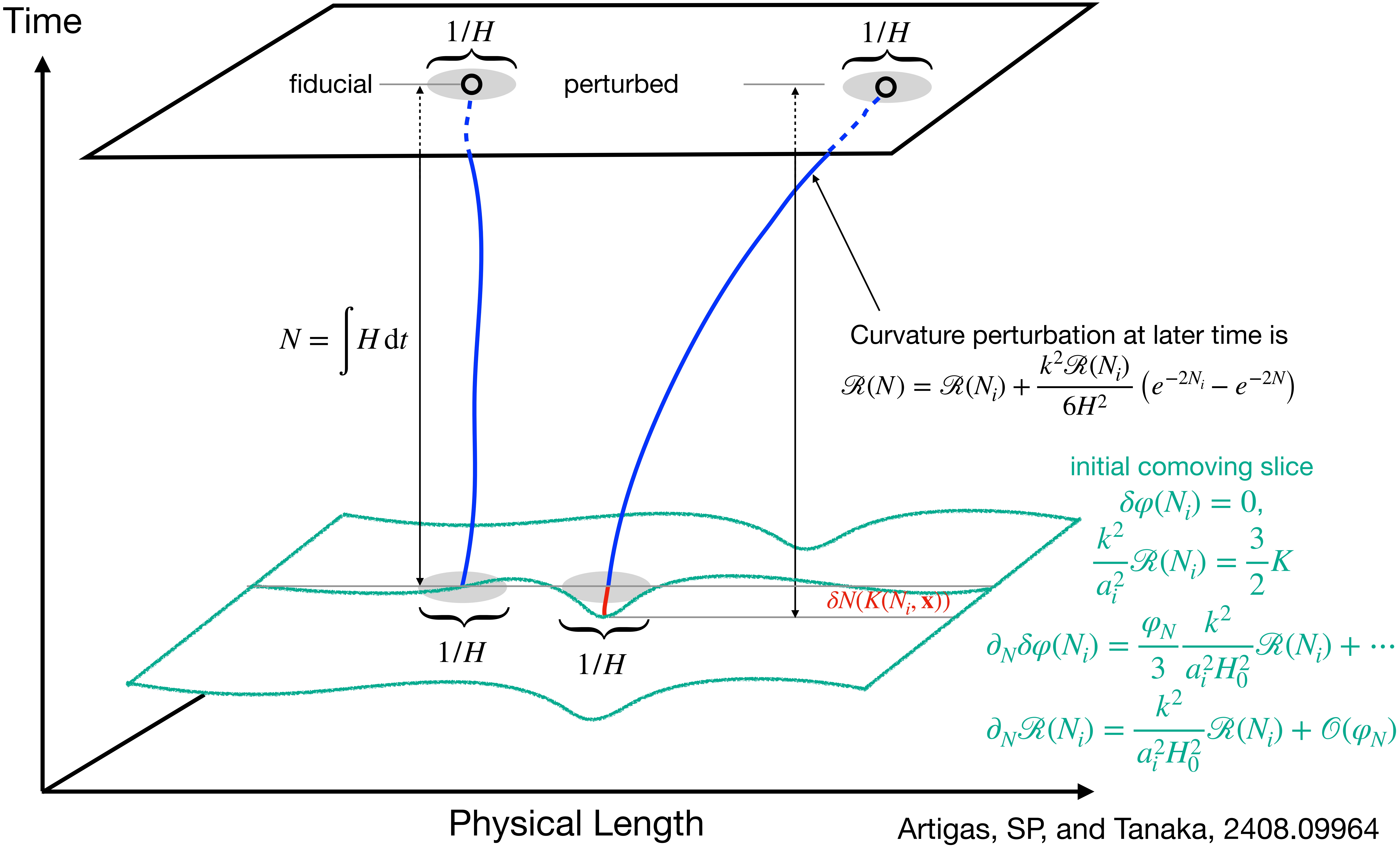
Artigas, SP, and Tanaka, 2408.09964

δN formalism



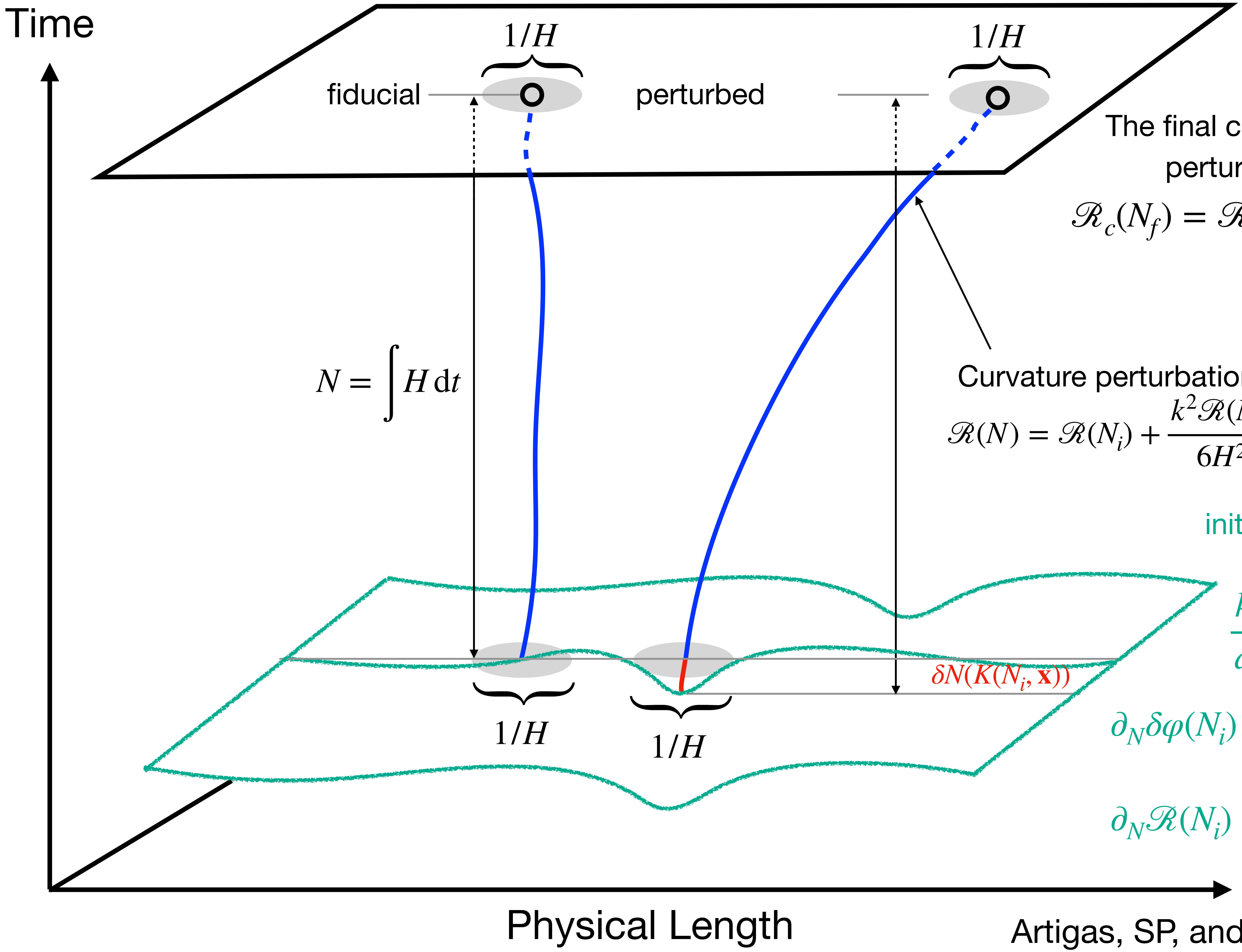
Artigas, SP, and Tanaka, 2408.09964

δN formalism



Artigas, SP, and Tanaka, 2408.09964

δN formalism



The final comoving curvature perturbation at N_f is

$$\mathcal{R}_c(N_f) = \mathcal{R}(N_f) + \delta N(K(N_i, \mathbf{x}))$$

Curvature perturbation at later time is

$$\mathcal{R}(N) = \mathcal{R}(N_i) + \frac{k^2 \mathcal{R}(N_i)}{6H^2}\left(e^{-2N_i} - e^{-2N}\right)$$

initial comoving slice

$$\delta\varphi(N_i) = 0,$$

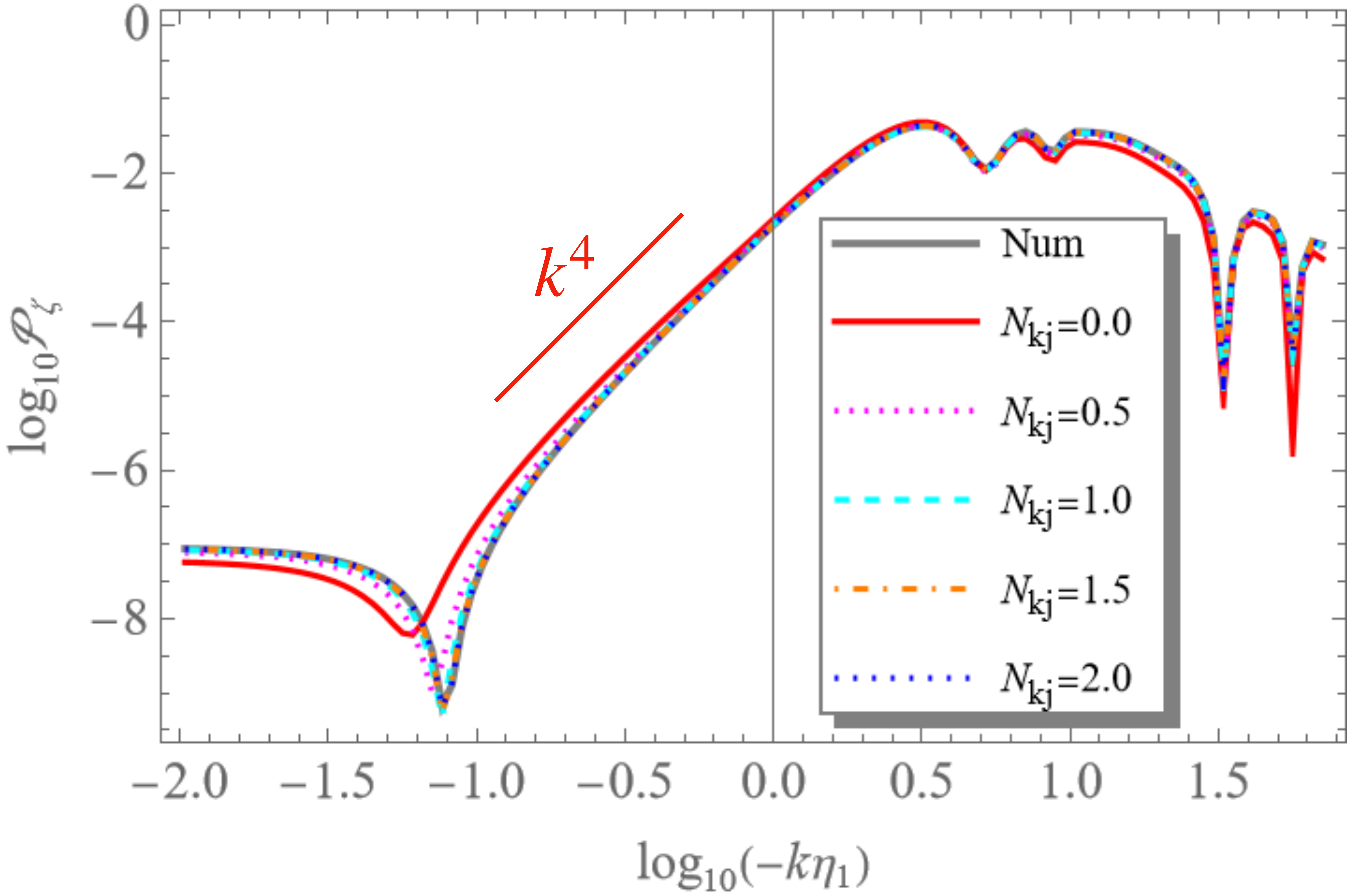
$$\frac{k^2}{a_i^2}\mathcal{R}(N_i) = \frac{3}{2}K$$

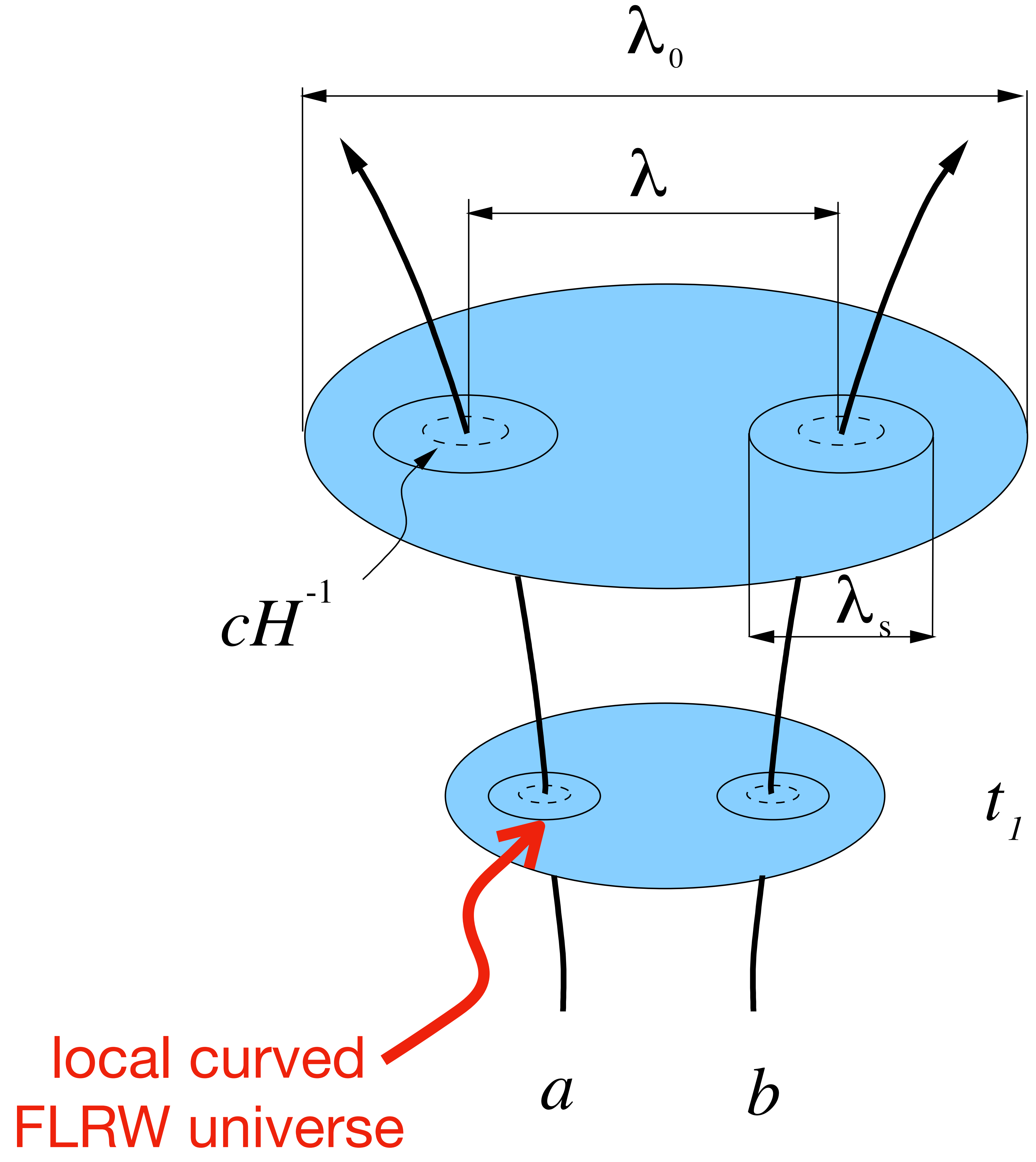
$$\partial_N \delta\varphi(N_i) = \frac{\varphi_N}{3}\frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \cdots$$

$$\partial_N \mathcal{R}(N_i) = \frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \mathcal{O}(\varphi_N)$$

Artigas, SP, and Tanaka, 2408.09964

Separate Universe



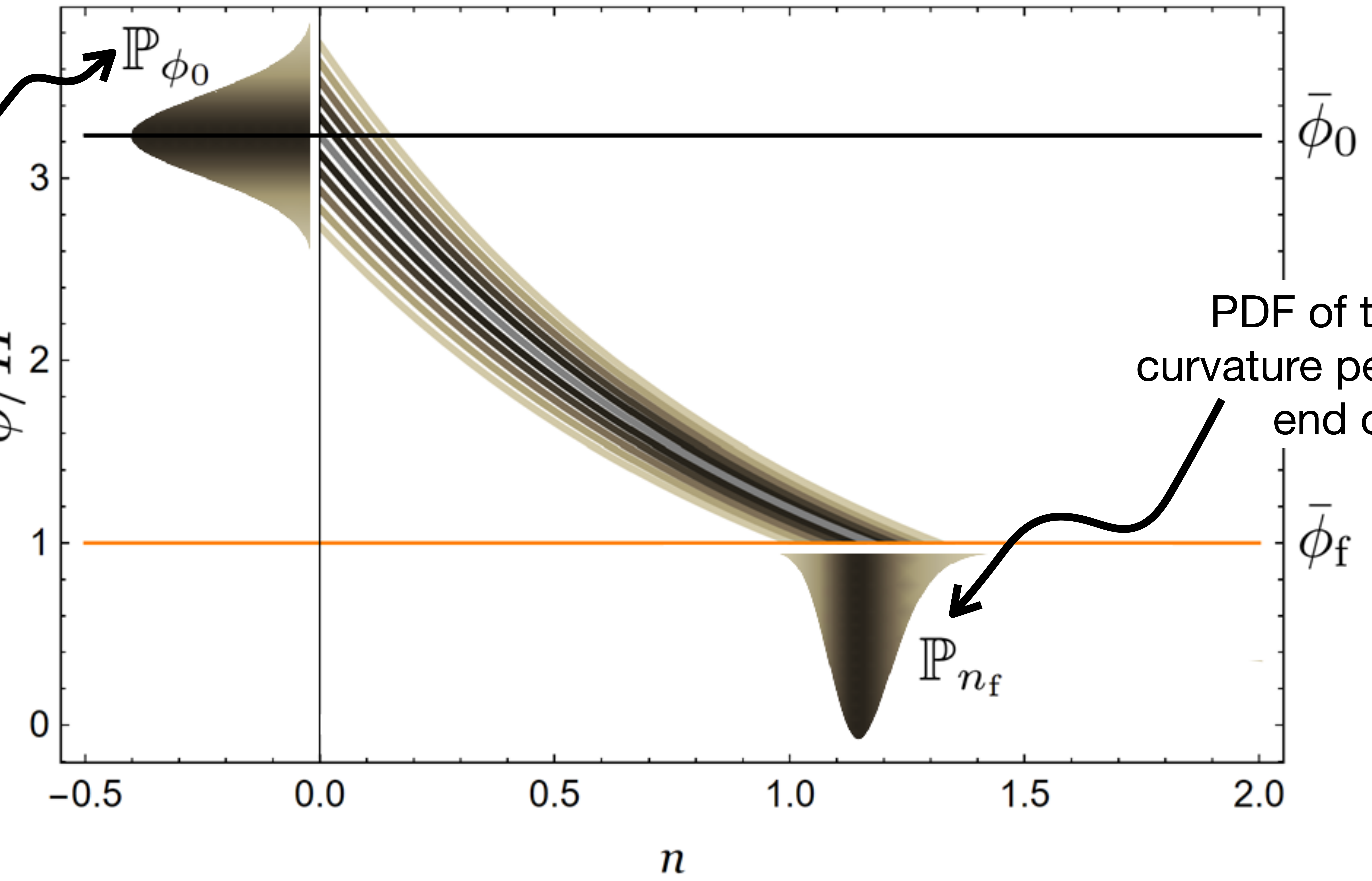


$$\mathcal{R}(N_e) = \mathcal{R}(N_i) + \delta N(k^2\mathcal{R}(N_i))$$

Artigas, SP, and Tanaka, 2408.09964

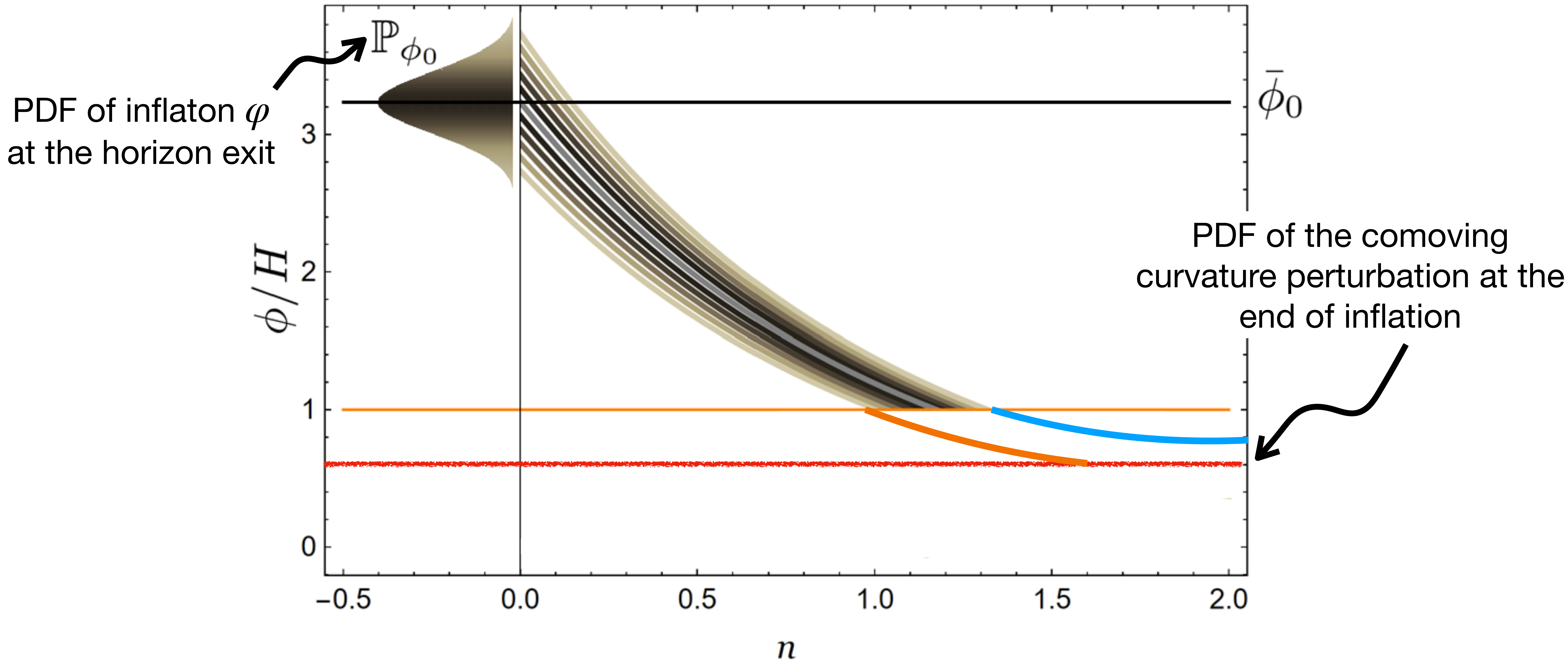
Ex2: Probability conservation

PDF of inflaton φ at the horizon exit

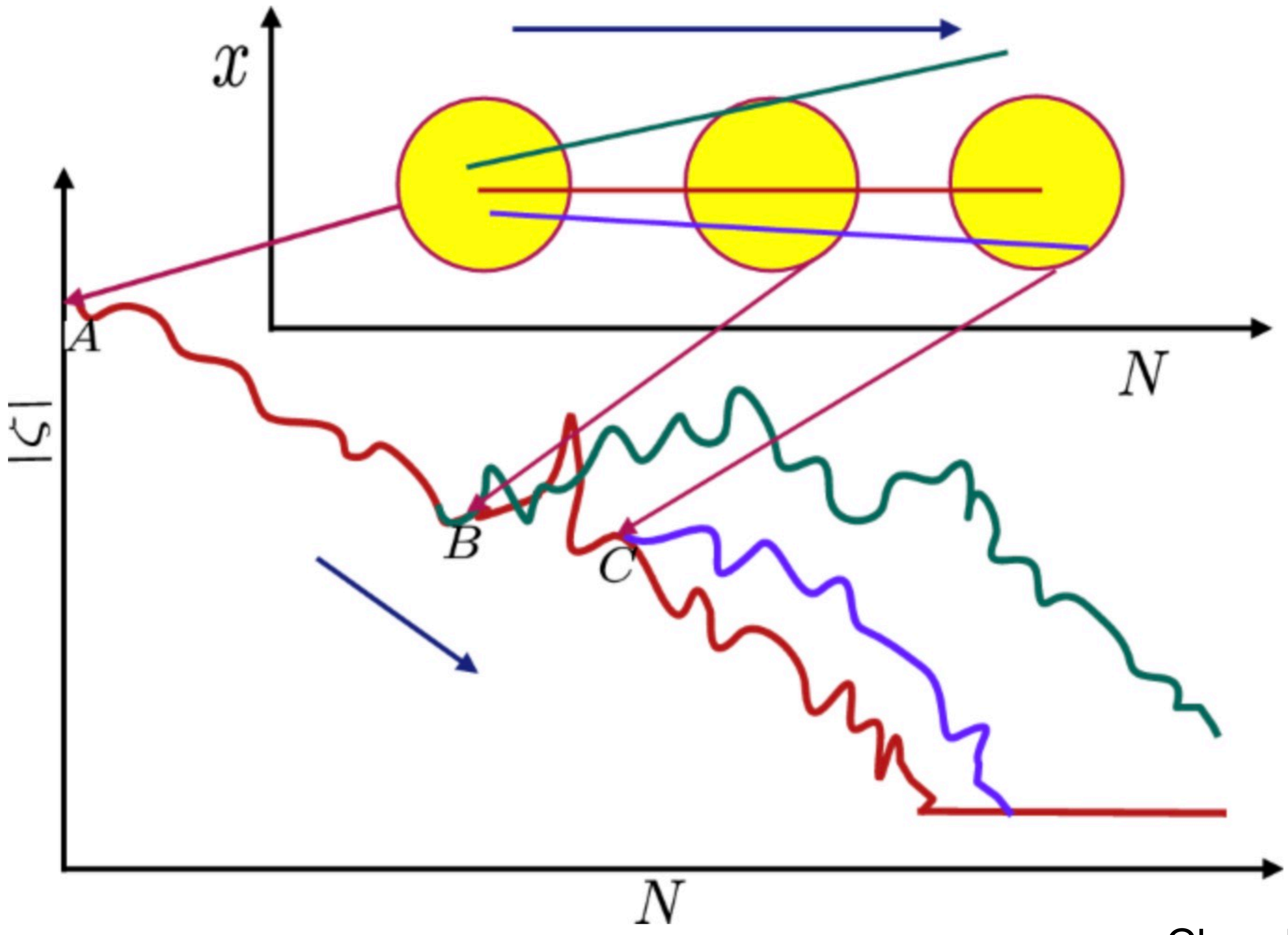


PDF of the comoving curvature perturbation at the end of inflation

Probability conservation

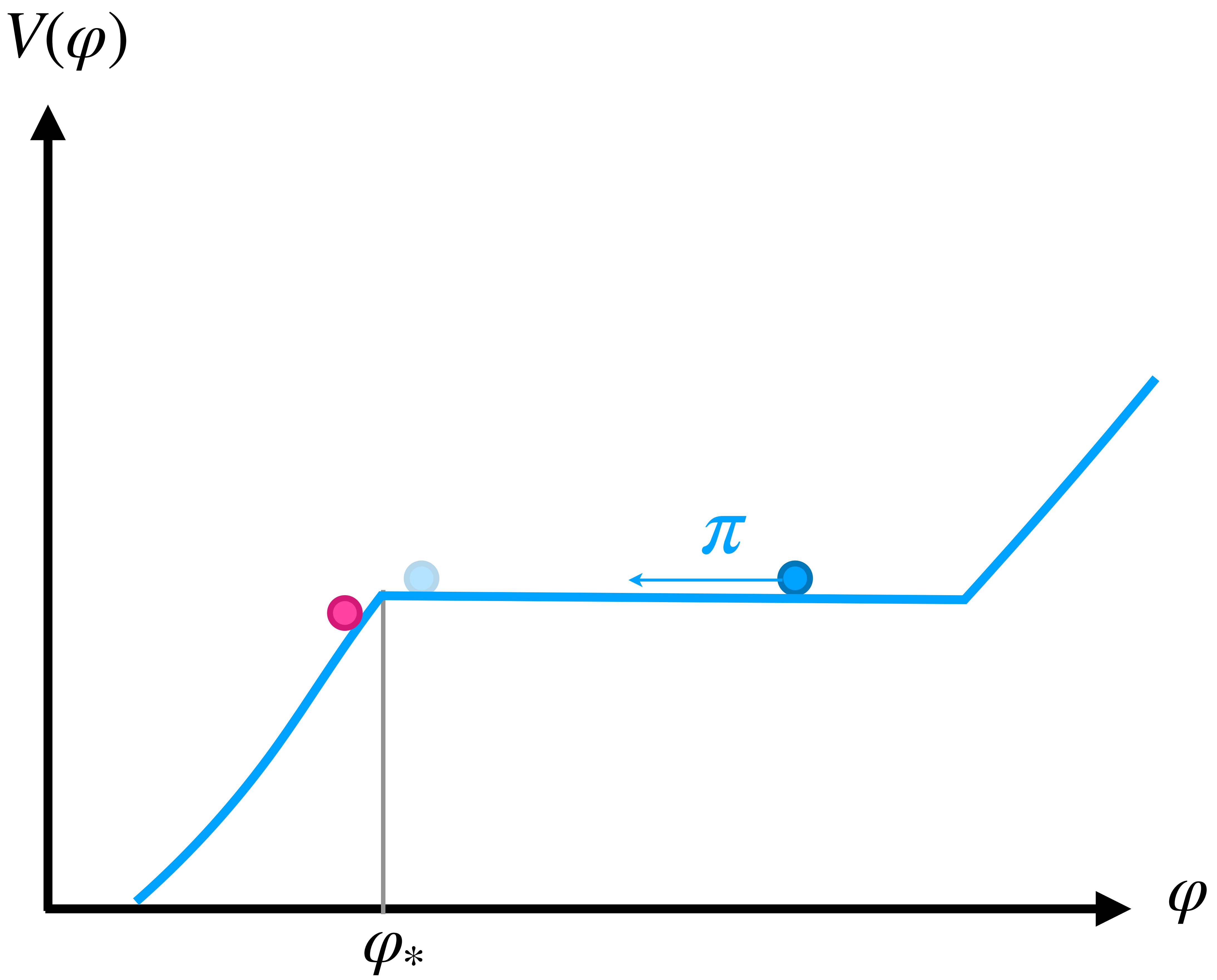


Probability conservation



Choudhury et al, 2403.13484

Ultra-slow-roll inflation



$$\mathcal{R} = -\frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

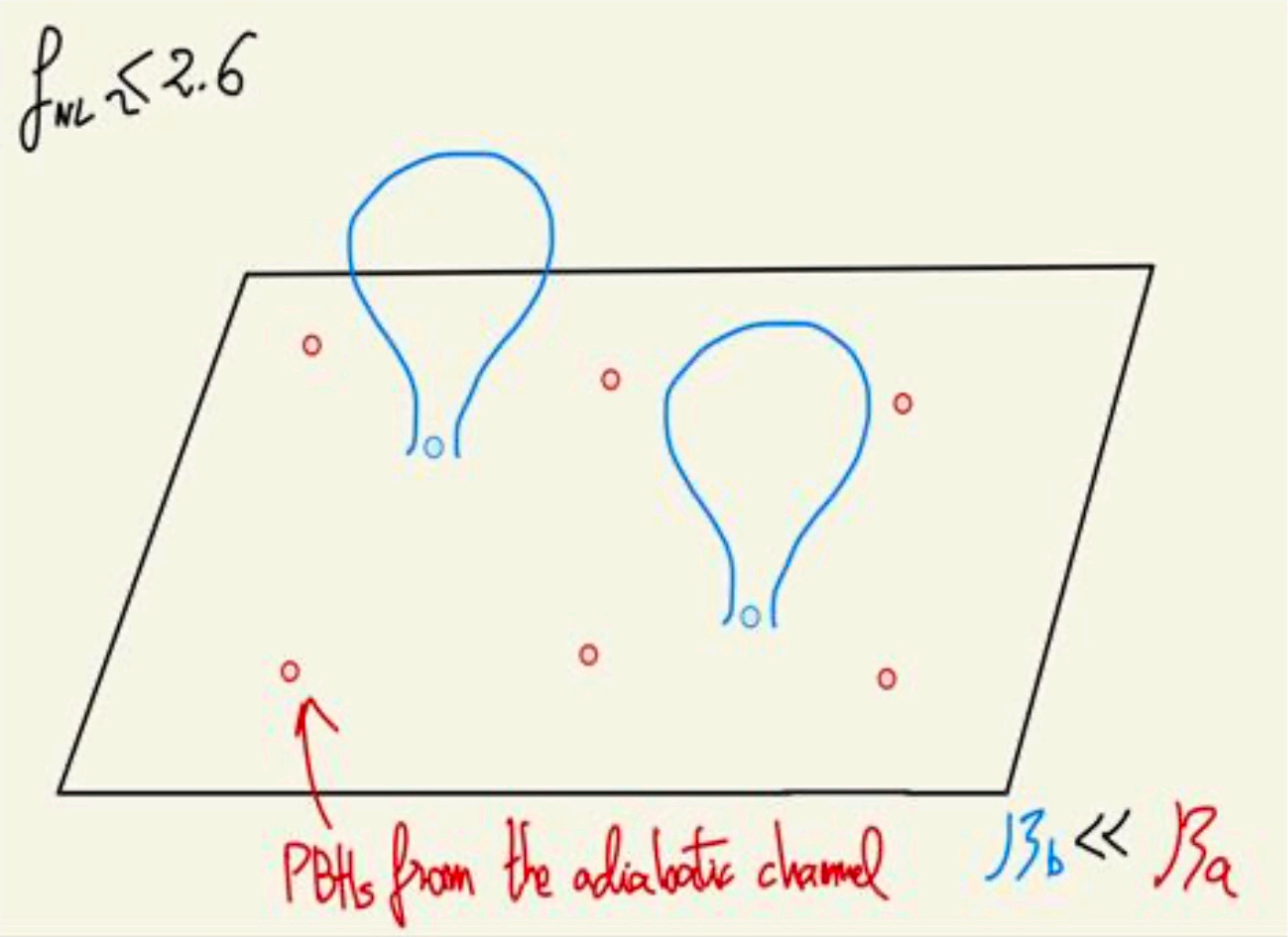
with $\delta\pi_*/\pi_* > 1$

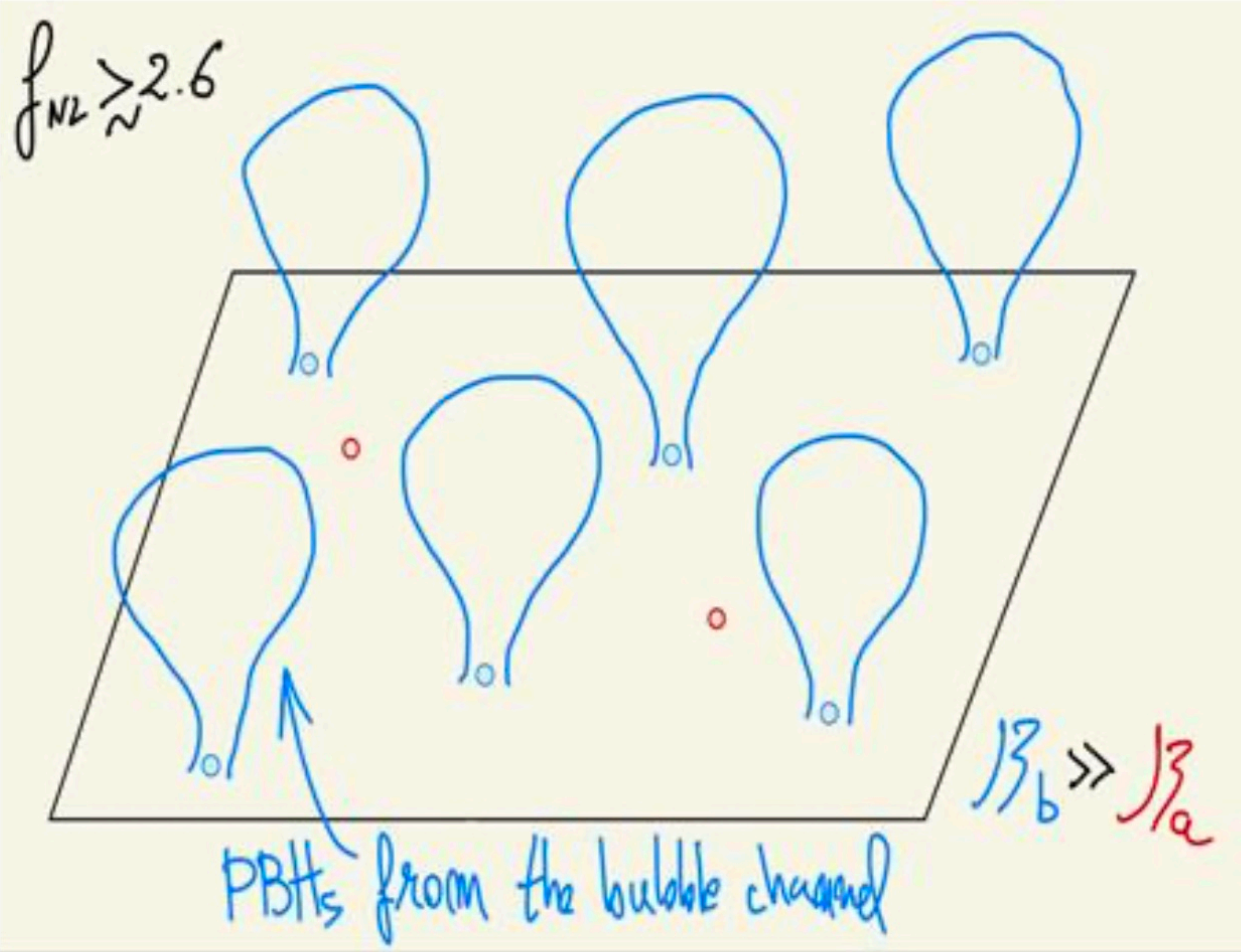
From Jaume Garriga's slide at Paris IHT

Qualitative picture

Fluctuations type I-> dominant contribution

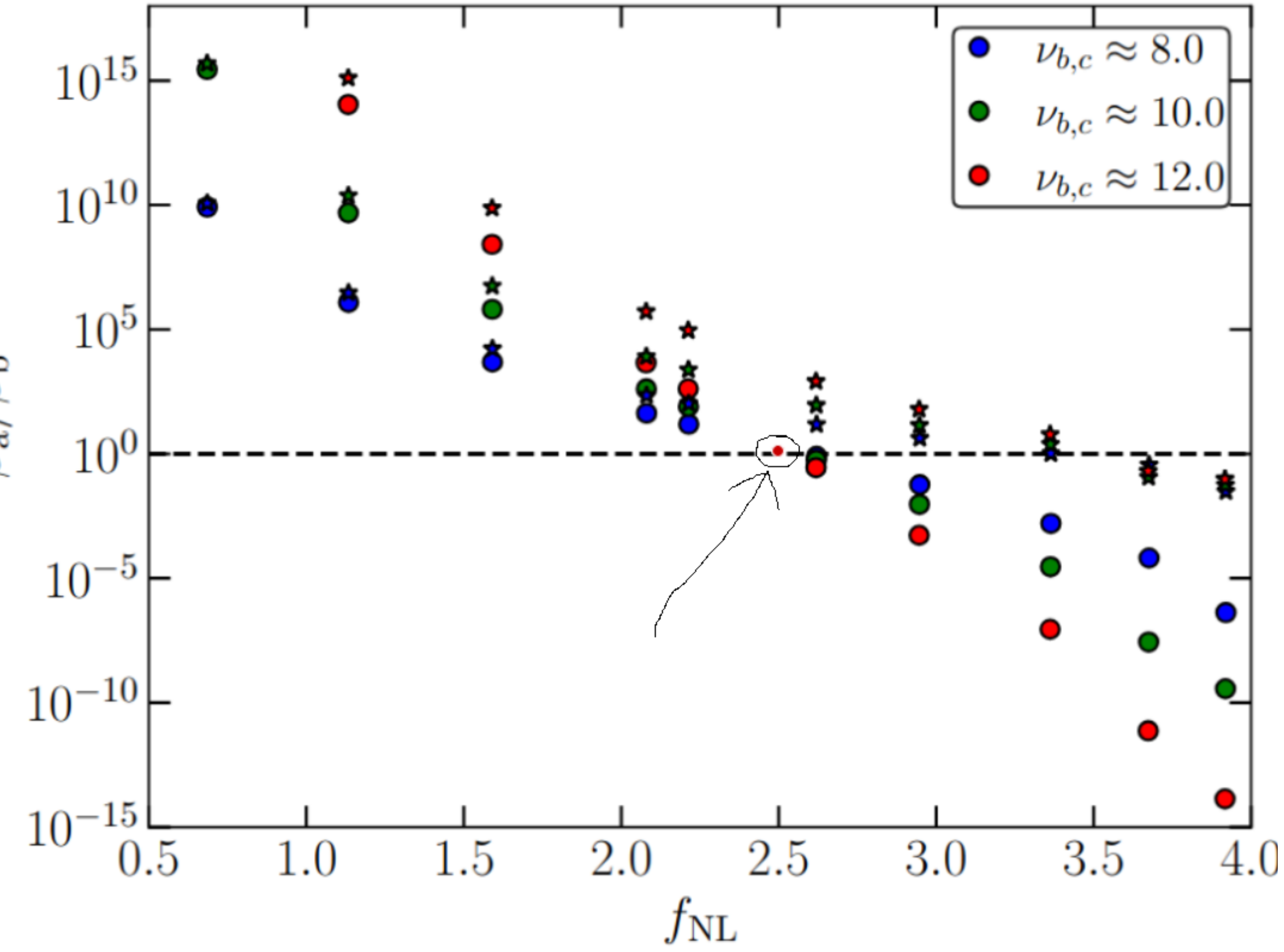
Fluctuations type II-> dominant contribution

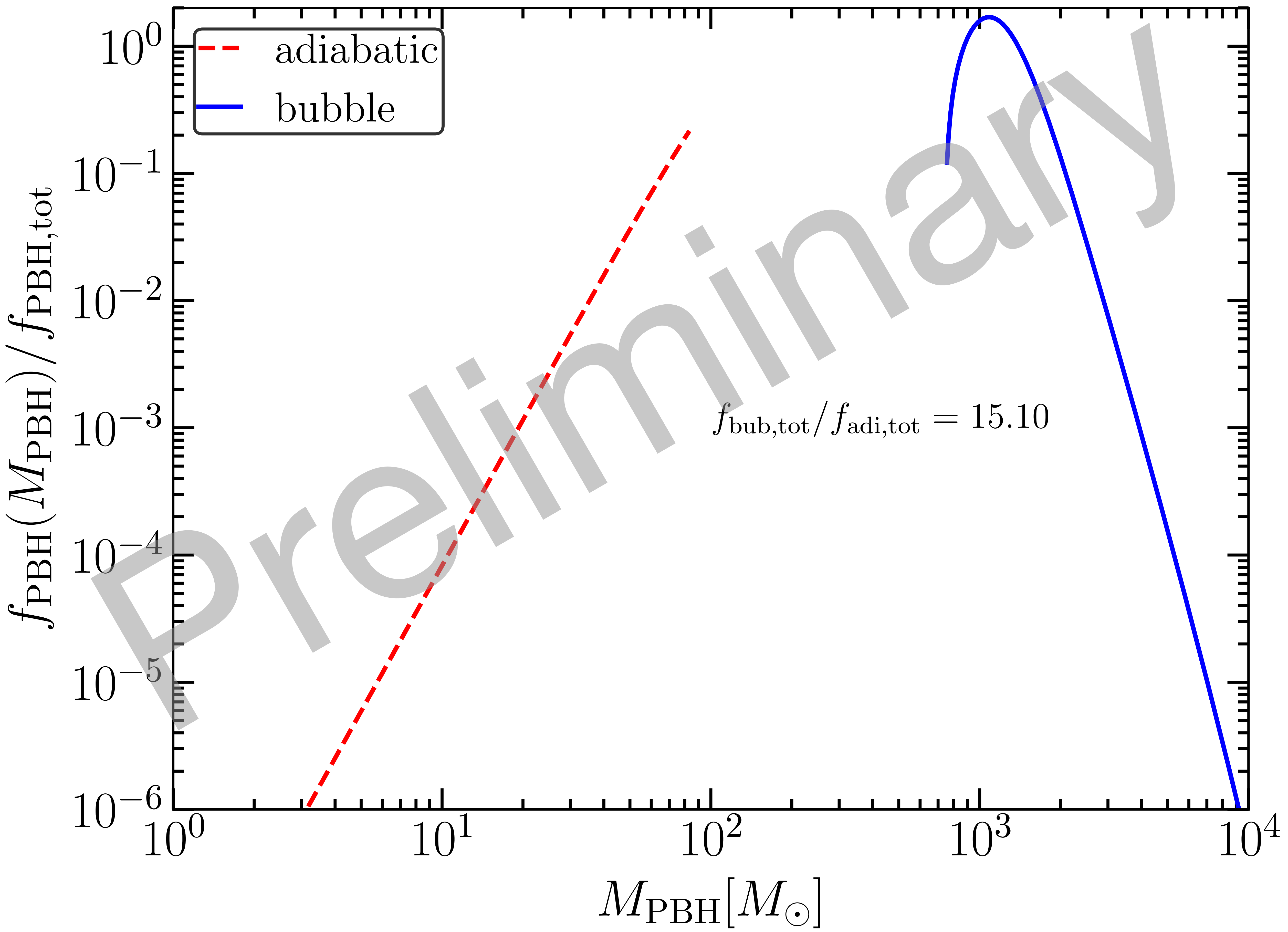




Escriva, Atal, Garriga, 2306.09990

Probability conservation





Escriva, Garriga, Pi, in prep

Conclusion

- In USR, the curvature perturbation are enhanced on superhorizon scales, which usually changes non-Gaussianity significantly.
- The NG generated on superhorizon scales can be calculated by δN formalism. Such a NG depends crucially on the boundary conditions.
- Original δN based on separate-universe approach fails to deal with decaying mode. We proposed the extended δN by considering K as the random variable.