Non-Gaussianities in the PBH Formation and Induced GWs

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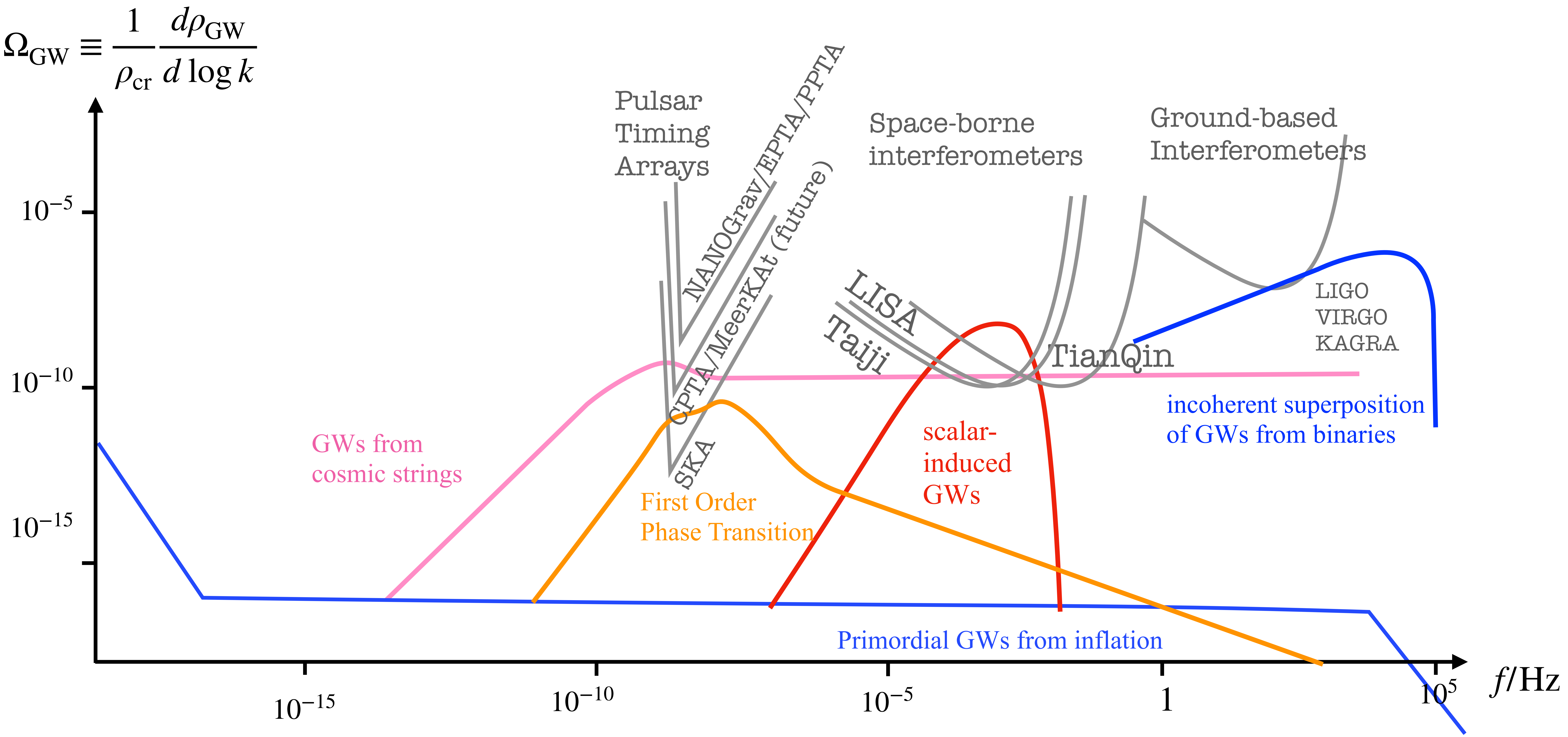
CONTENT

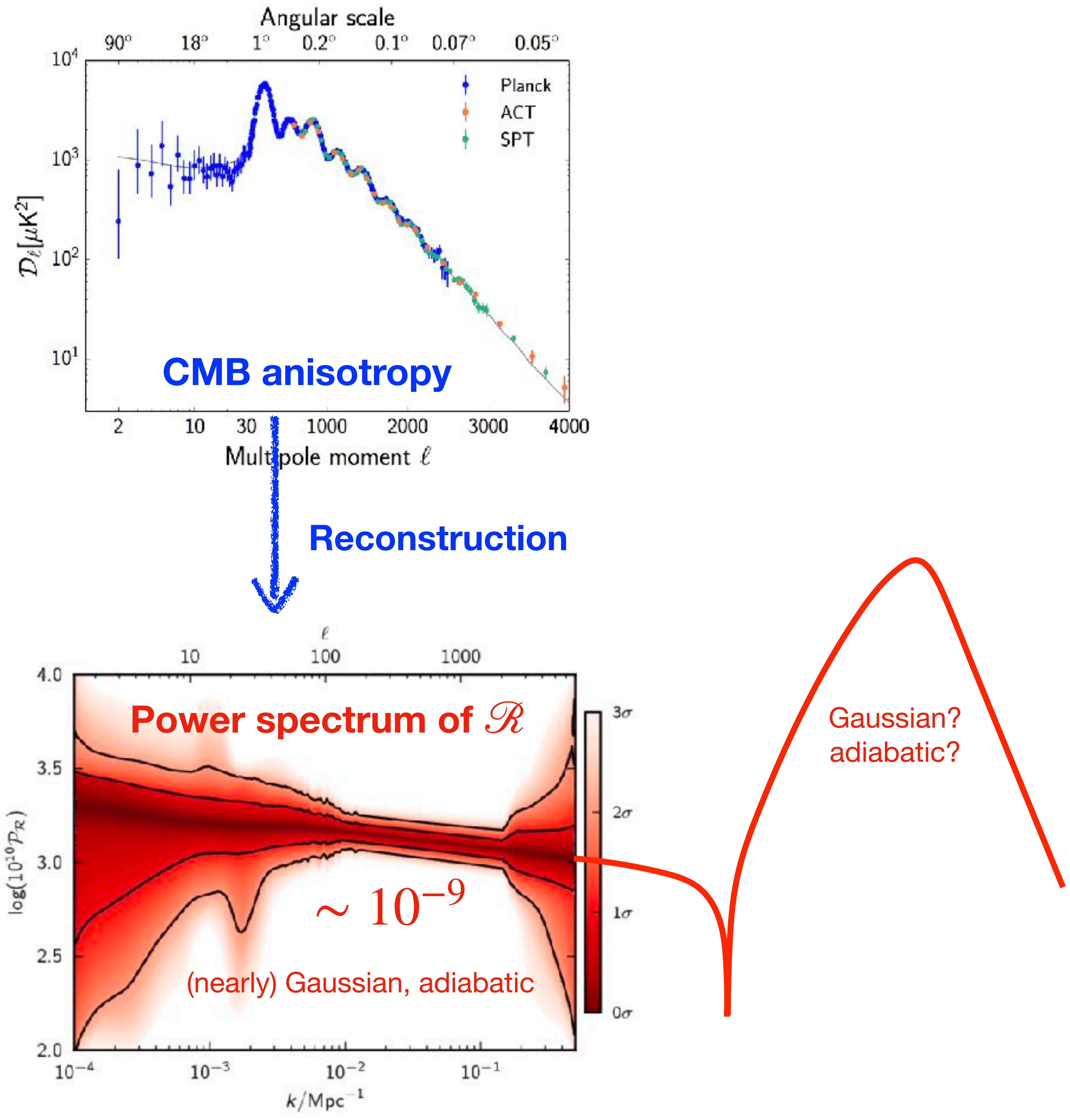
- Introduction: PBH and IGW
- Primordial NG of the curvature perturbation
- Applications
- Summary

Introduction:

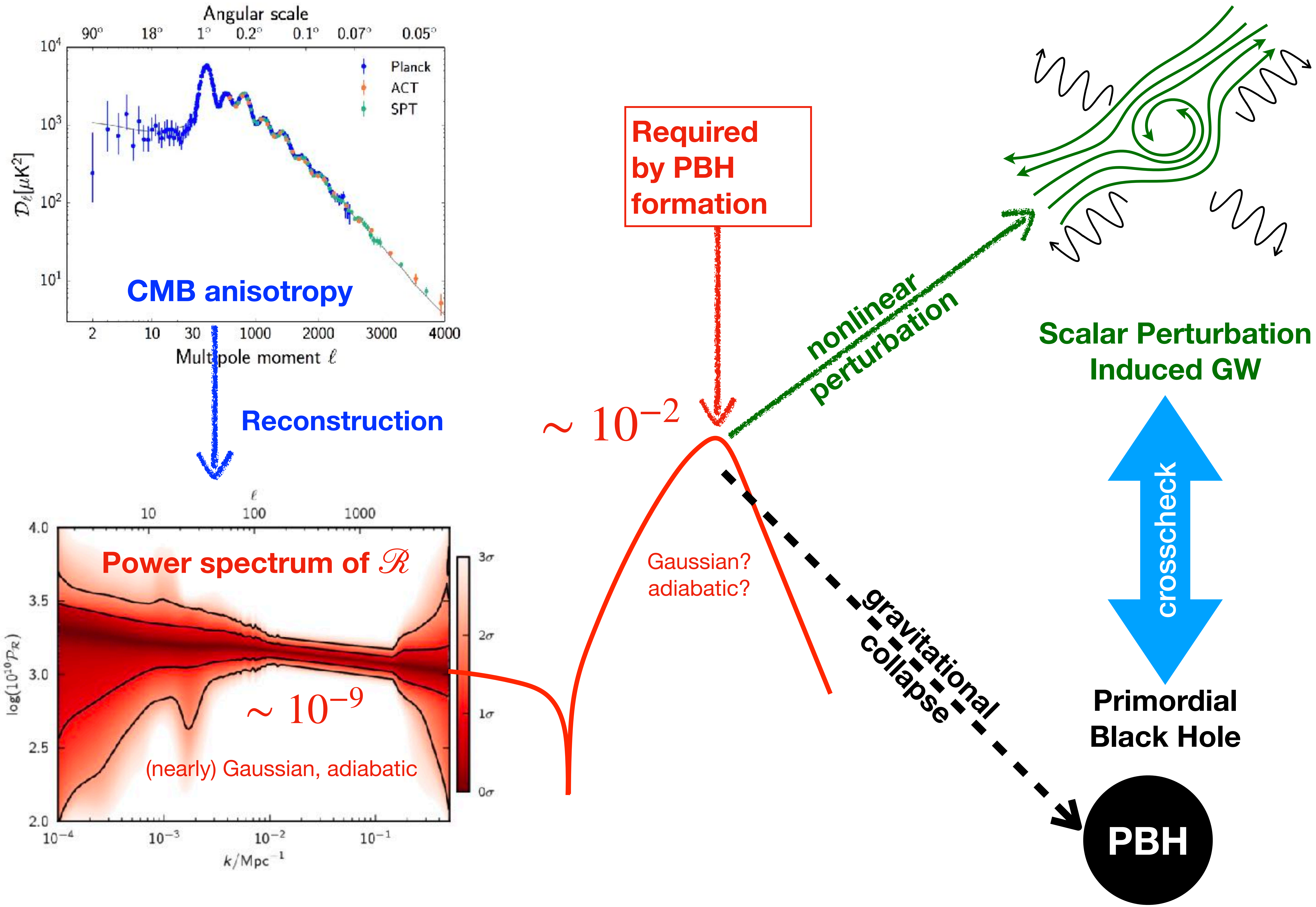
Primordial Black Hole and Scalar Induced Gravitational Waves

Possible SGWB Sources





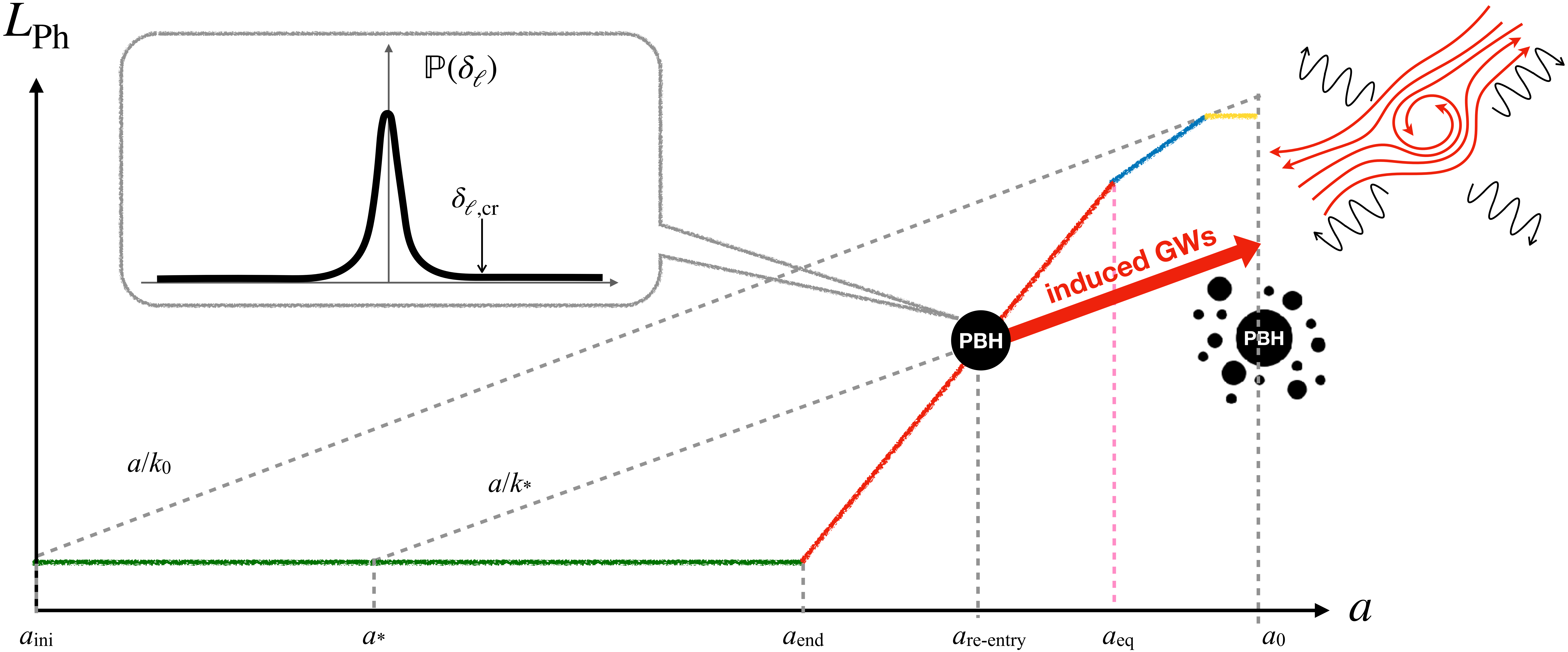
Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
CMB anisotropy
Multipole moment ℓ
Reconstruction
ℓ
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
$\sim 10^{-9}$
(nearly) Gaussian, adiabatic
k/Mpc^{-1}
3σ
2σ
1σ
0σ
Gaussian?
adiabatic?



Matarrese et al, PRD 47, 1311;
PRL 72, 320; PRD 58, 043504
Ananda et al, gr-qc/0612013
Bauman et al, hep-th/0703290

Zeldovich & Novikov 1966
Hawking 1971
Carr & Hawking 1974

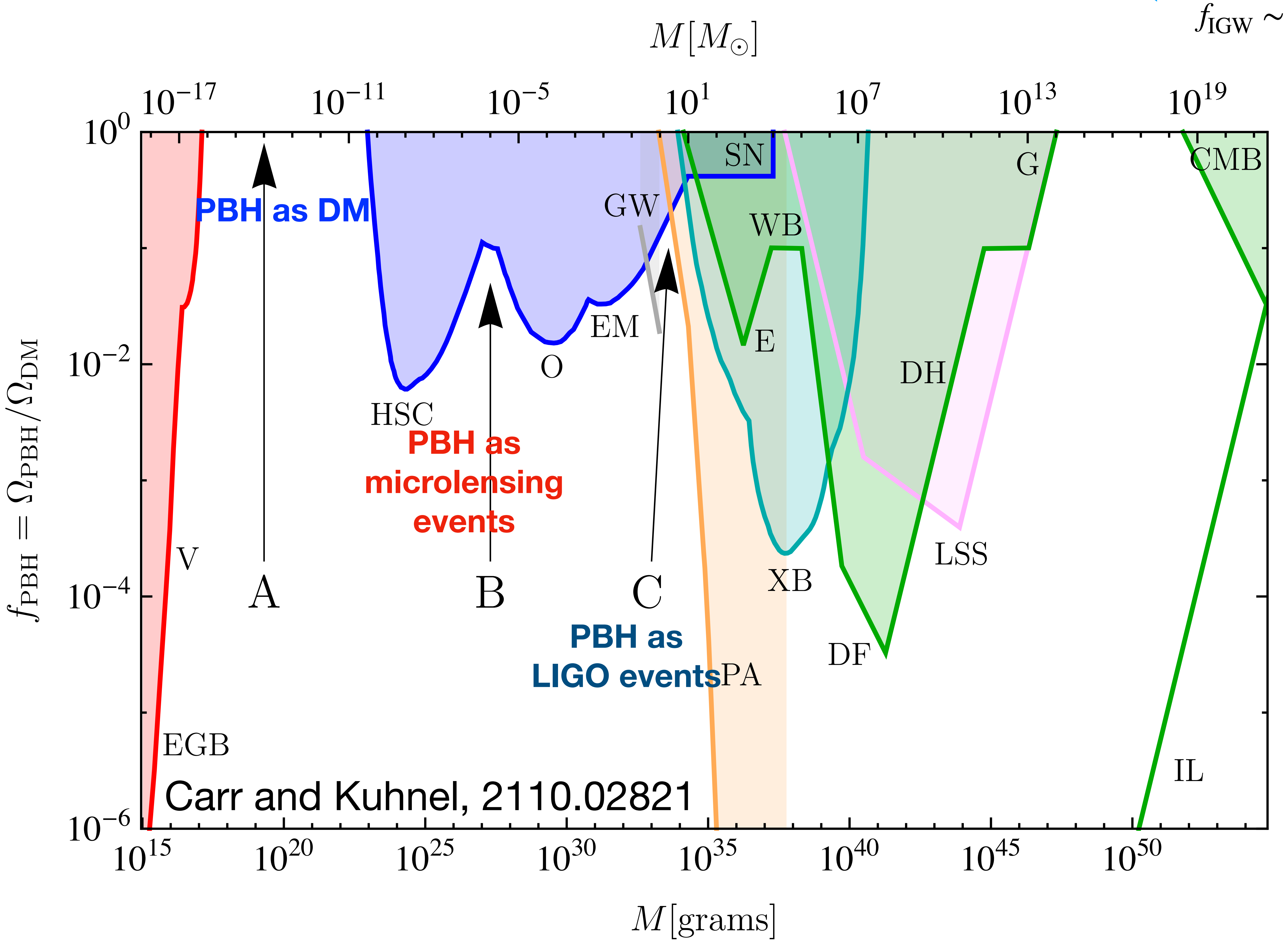
PBH-IGW crosscheck



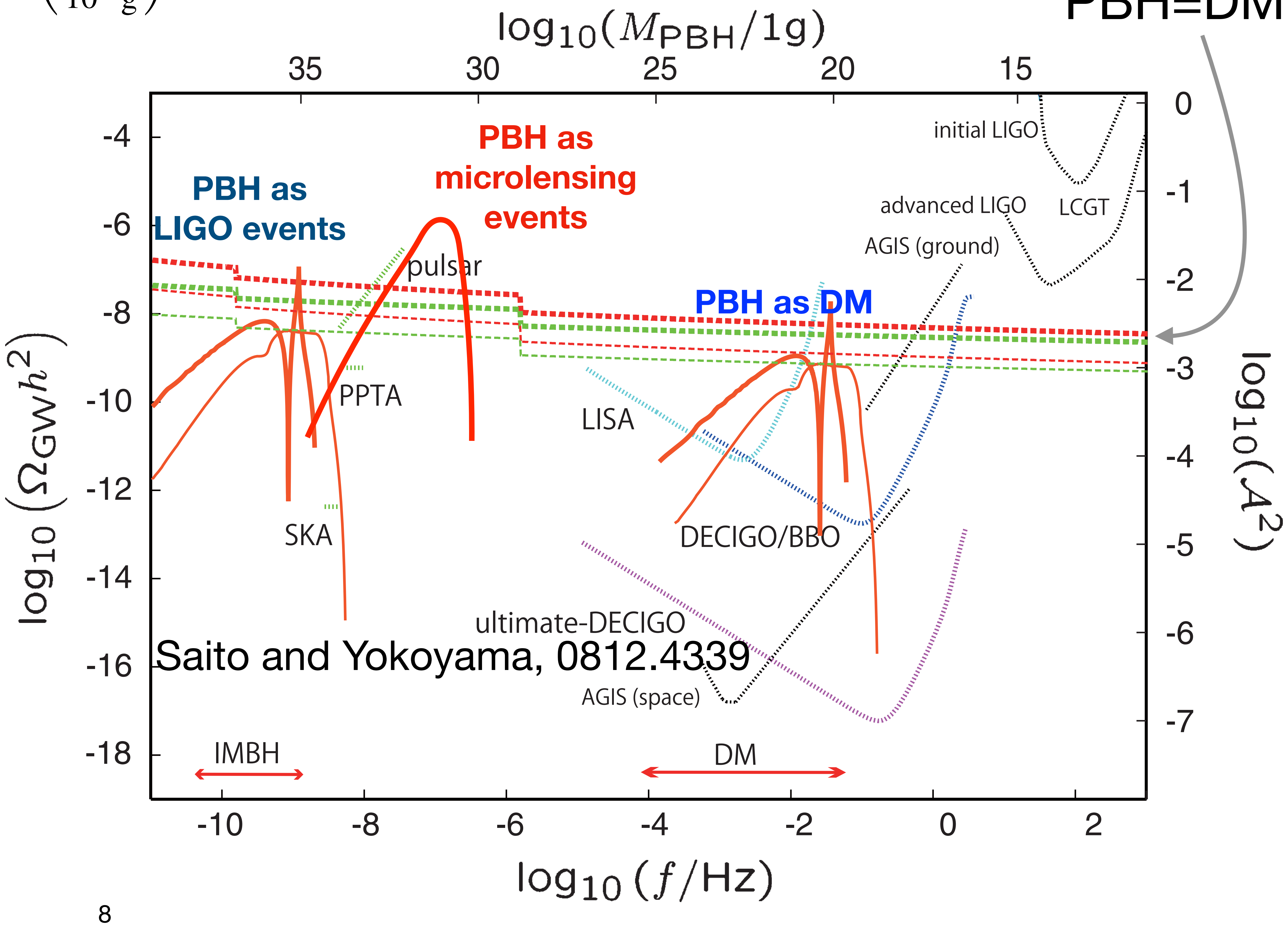
PBH-IGW crosscheck

PBH constraints ⟷ crosscheck ⟷ IGW constraints

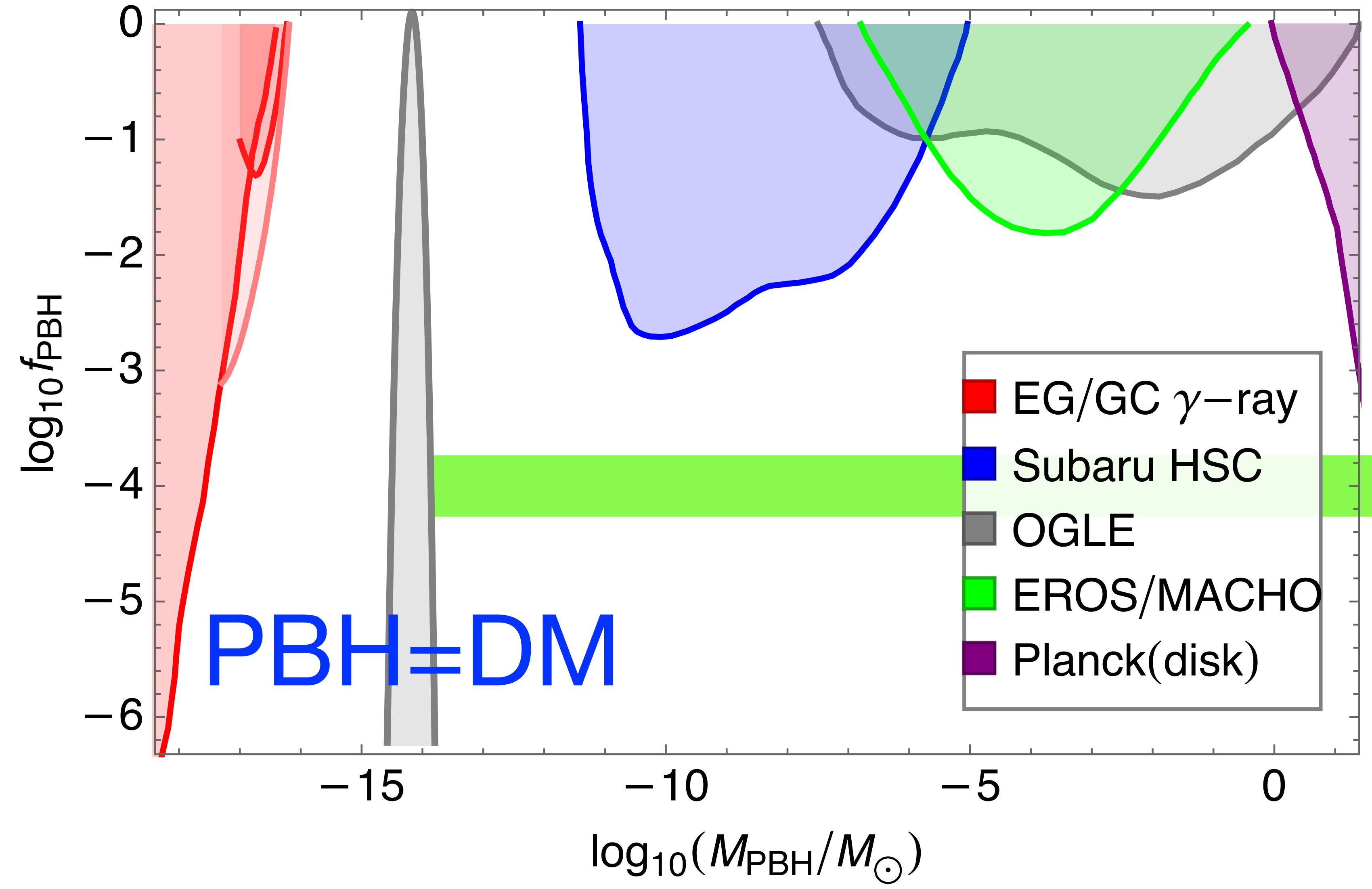
$$f_{\rm IGW} \sim 3{\rm Hz}\left(\frac{M_{\rm PBH}}{10^{16}{\rm g}}\right)^{-\frac{1}{2}}$$

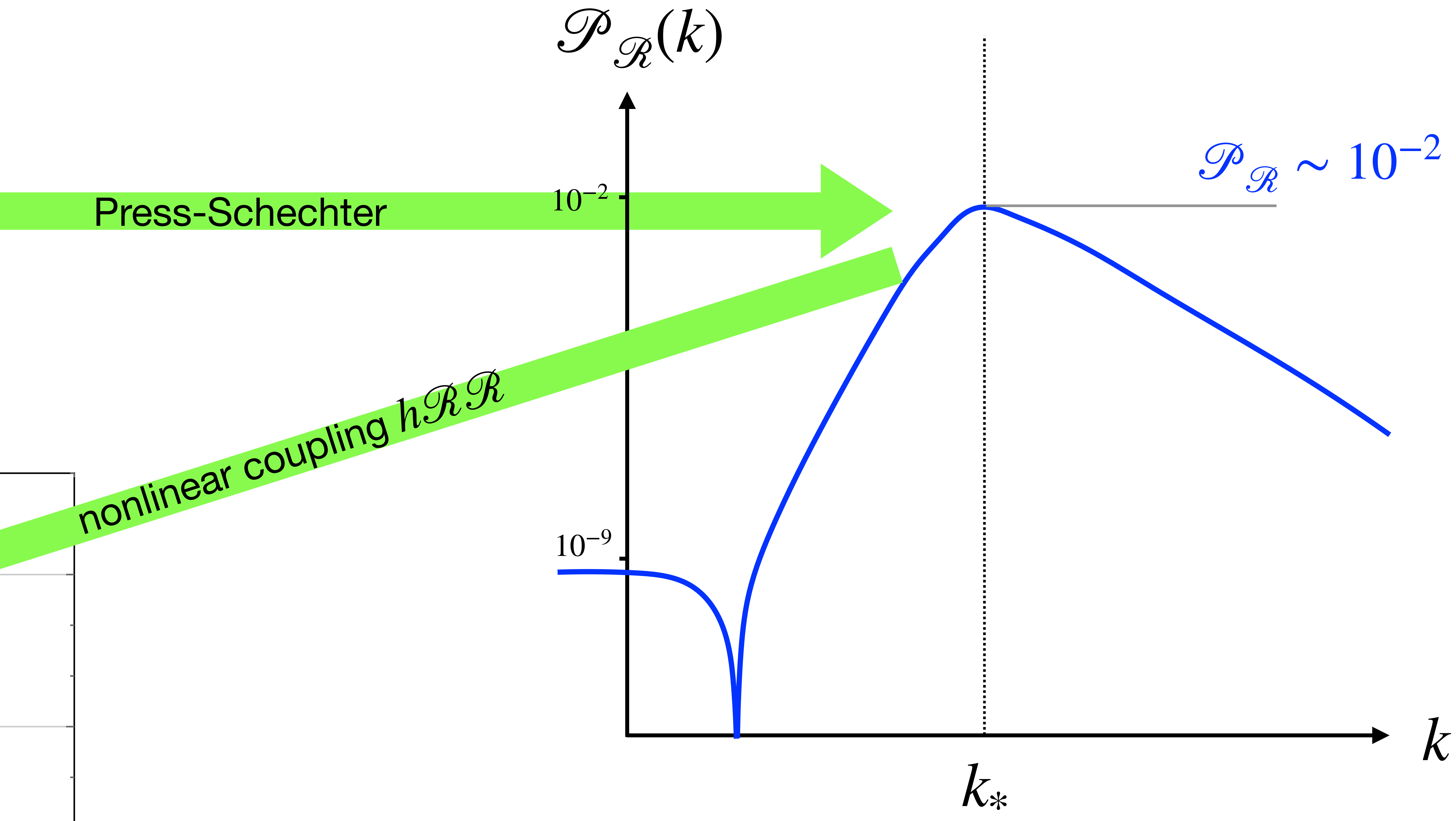


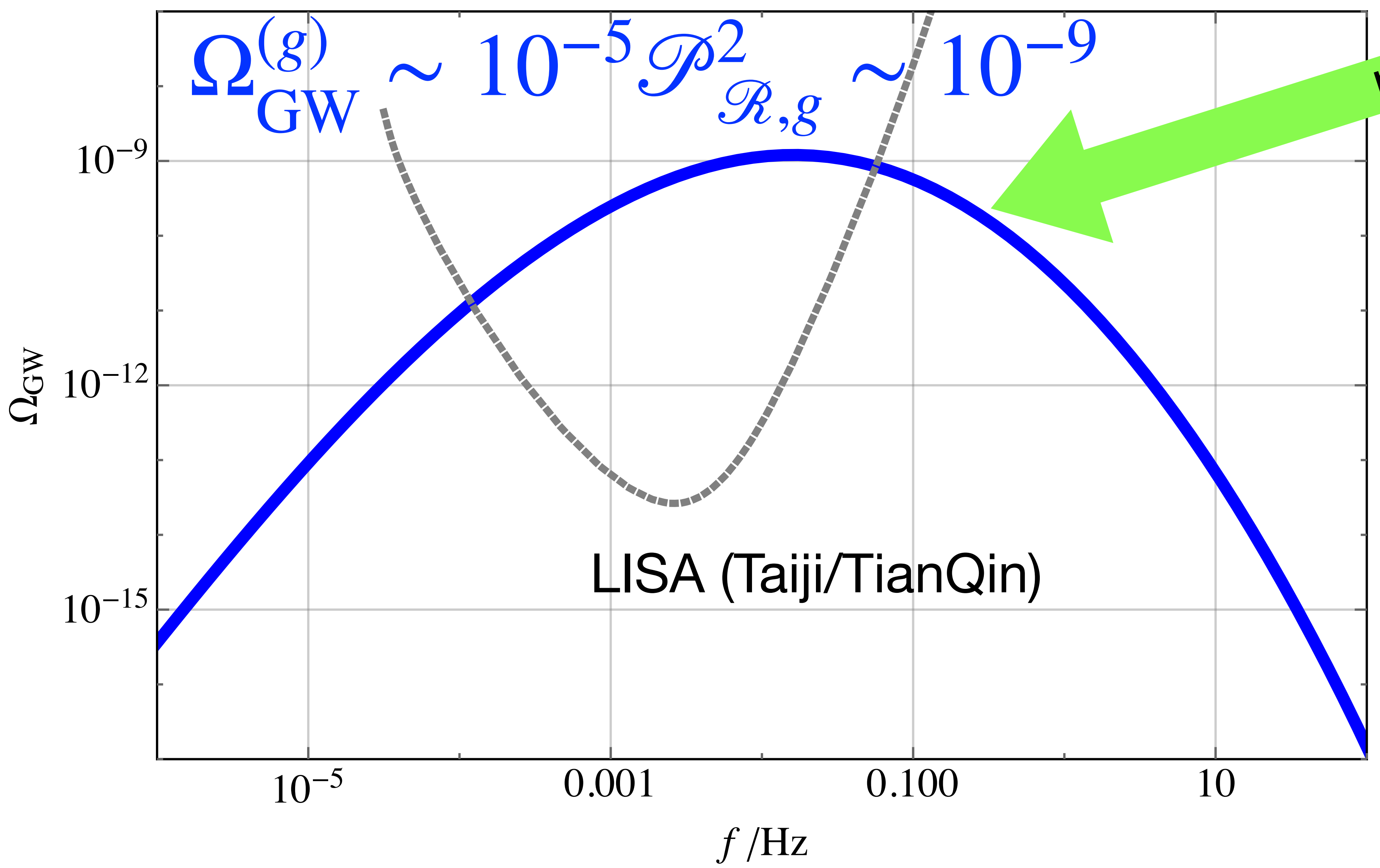
PBH=DM



PBH-IGW crosscheck

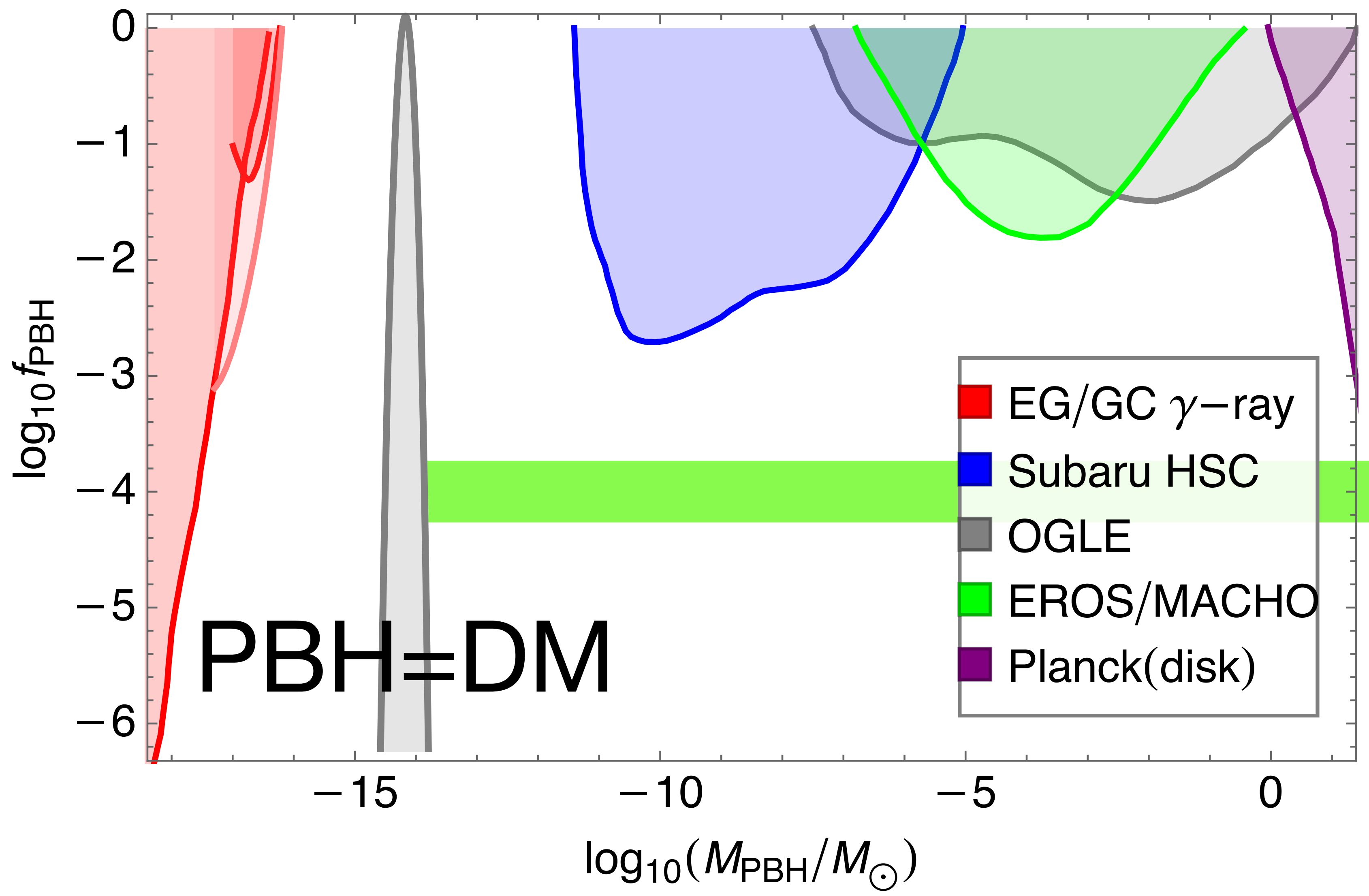






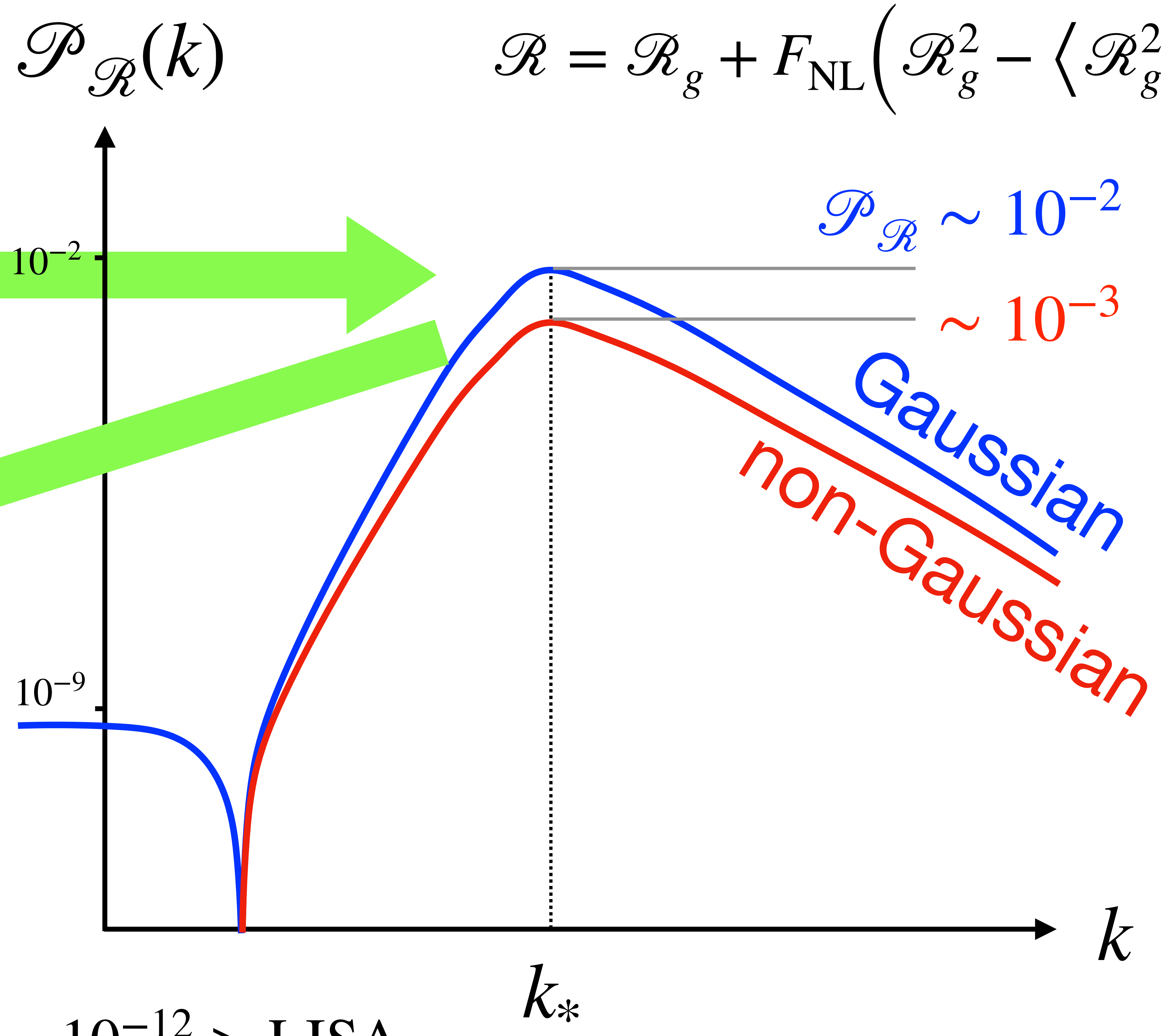
Saito & Yokoyama 0812.4339; 0912.5317
Bugaev & Klimai 0908.0664; 1012.4697
Escriva et al, 2211.05767

Including non-Gaussianity



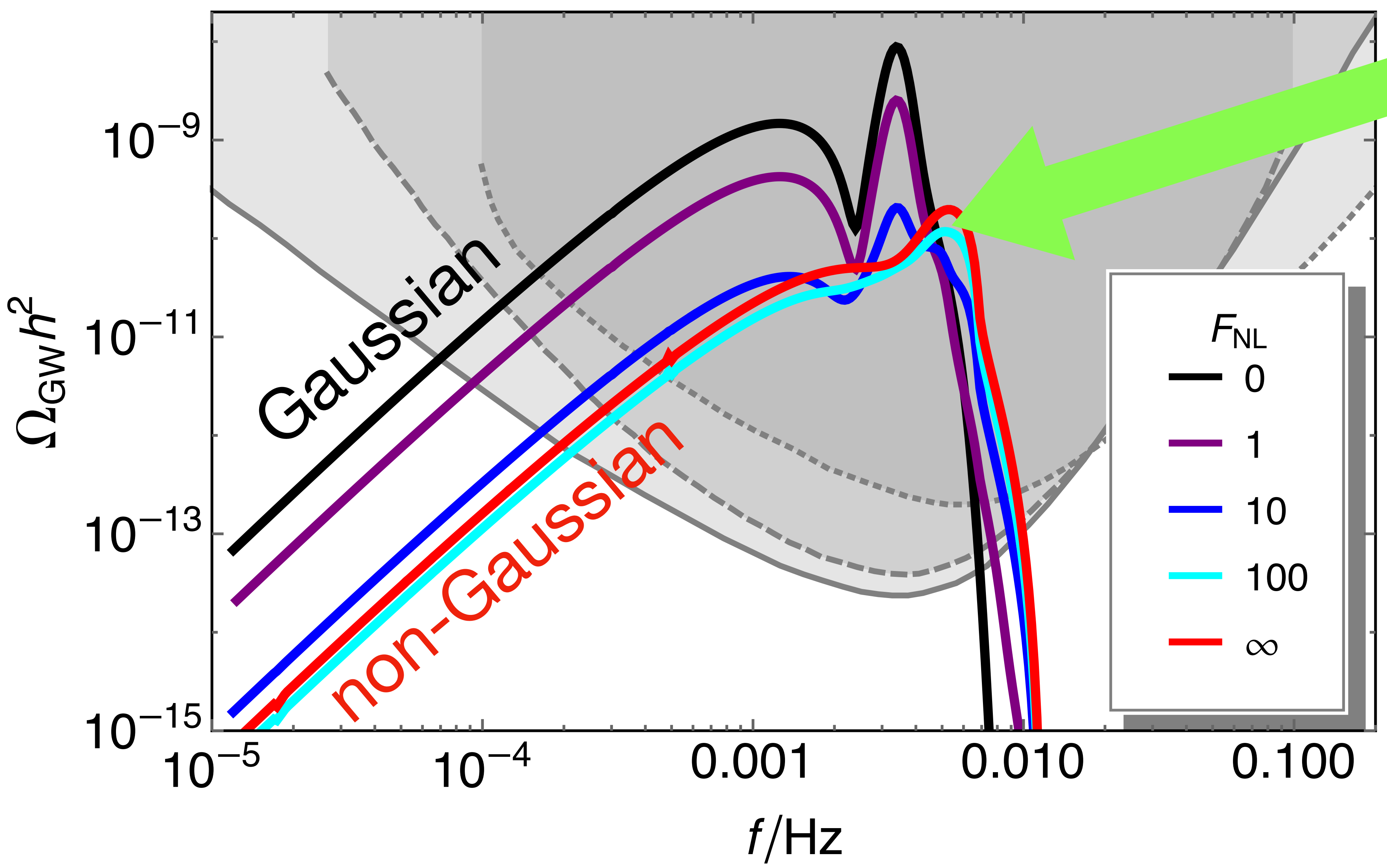
Press-Schechter

$$\mathcal{R} = \mathcal{R}_g + F_{\rm NL}\left(\mathcal{R}_g^2 - \langle \mathcal{R}_g^2 \rangle\right)$$



nonlinear coupling $h\mathcal{R}\mathcal{R}$

with $F_{\rm NL} > 0$



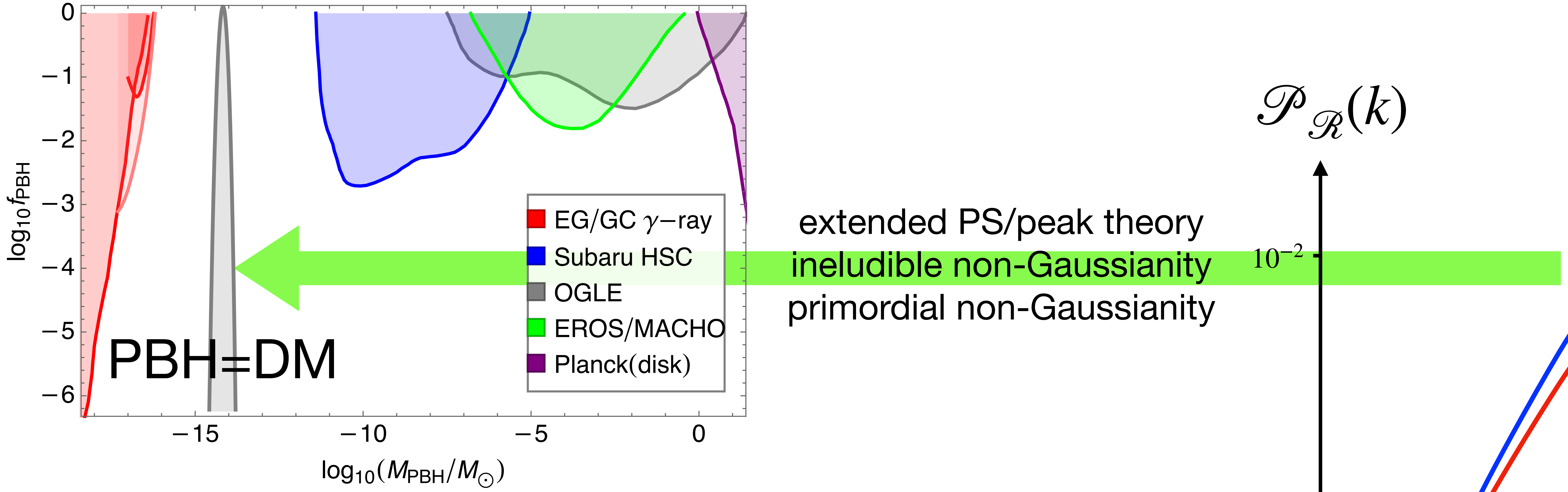
$F_{\rm NL} \mathcal{P}_\mathcal{R} \sim 10^{-2}$

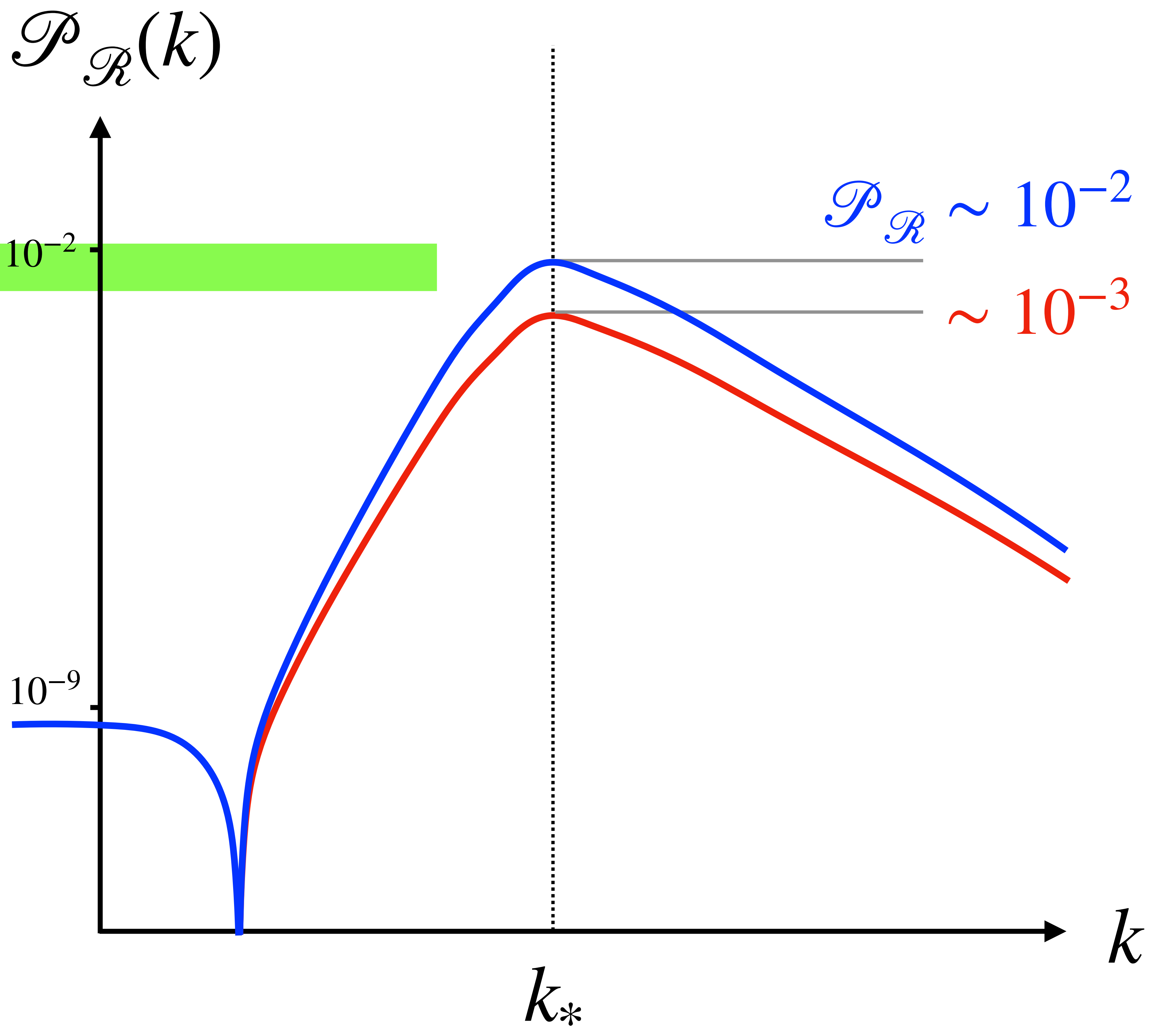
$\Omega_{\rm GW}^{\rm (NG)} \sim 10^{-5} F_{\rm NL}^4 \mathcal{P}_{\mathcal{R},g}^4 \sim 10^{-12} > {\rm LISA}$

LISA/Taiji/TianQin can probe PBH-DM.

Cai, SP, Sasaki, 1810.11000
Bartolo et al., 1810.12218

More non-Gaussianities



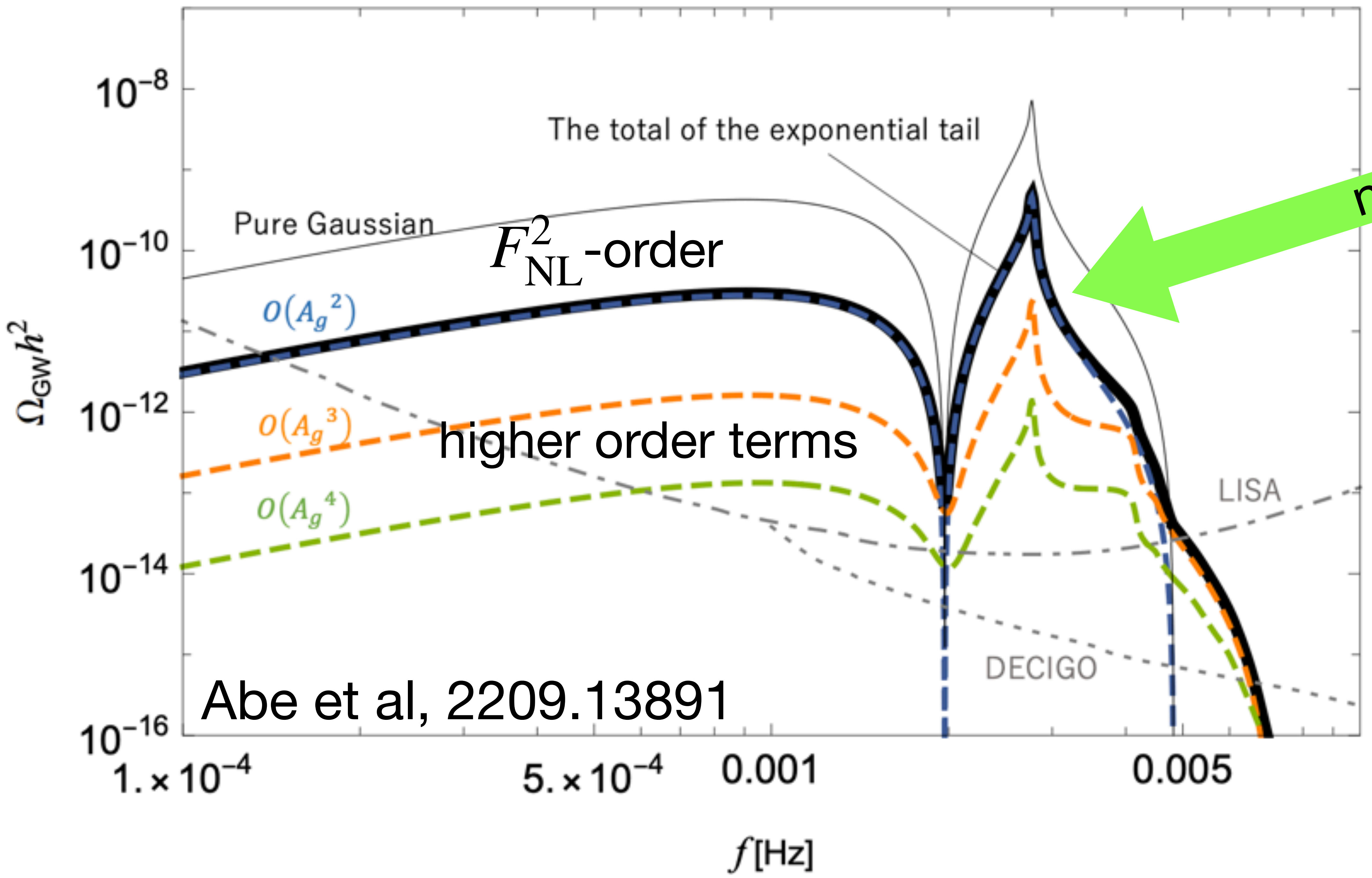


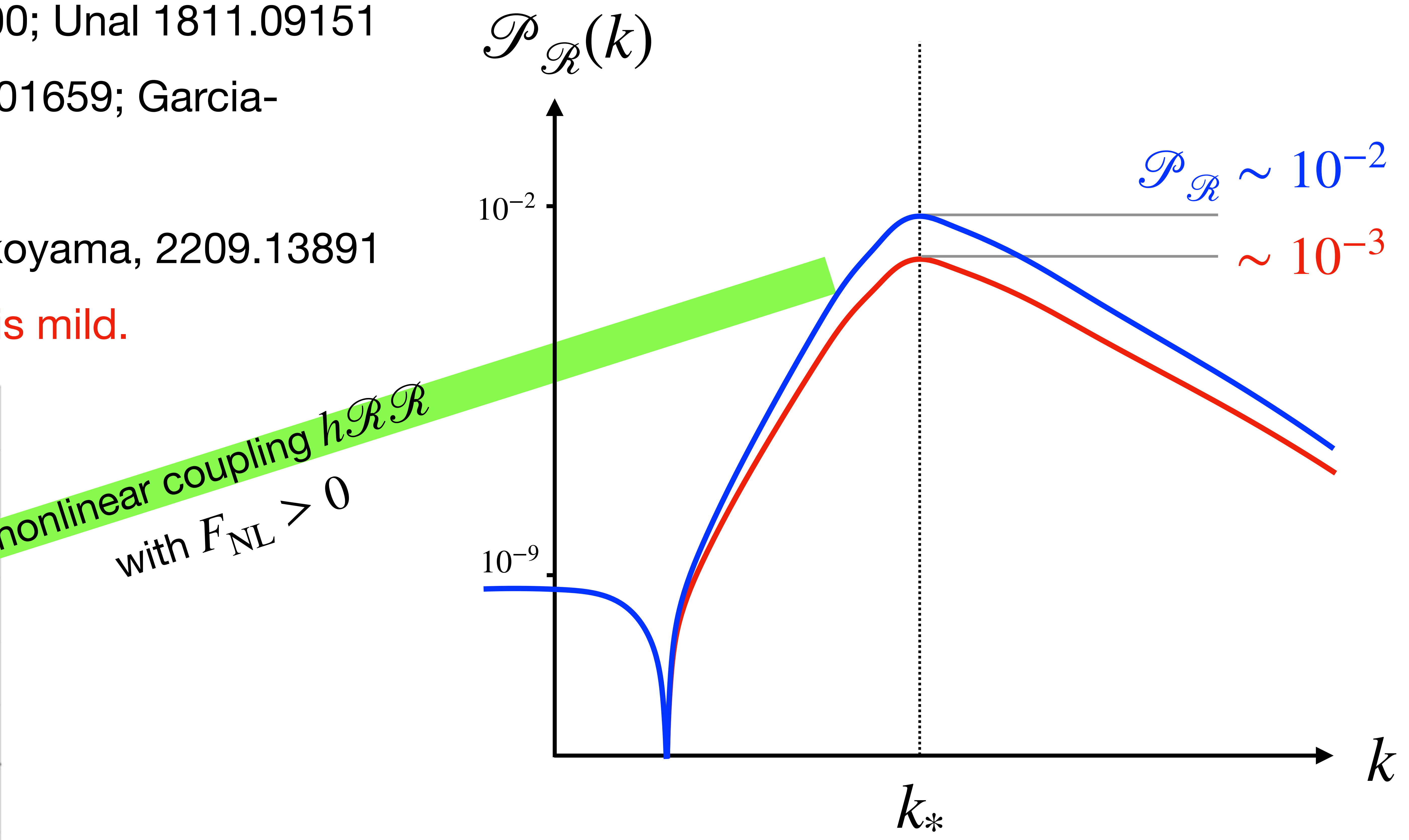
- Press-Schechter: Young & Byrnes 1307.4995; Young et al 1405.7023
- Extended Press-Schechter: Biagetti et al 2105.07810; Gow et al 2211.08348; Ferrante et al 2211.01728
- Peak theory: De Luca et al 1904.00970; Atal et al 1905.13202; Yoo et al 2008.02425; Kitajima et al 2109.00791; Escrivà et al 2202.01028; Germani & Sheth 1912.07072;
- Non-Gaussianity is important in calculating the PBH abundance.

SP, Sasaki, Takhistov, Jianing Wang, in prep.
Jianing's Talk 1130-B3-1

Non-Gaussianities in IGW

- Quadratic expansion: Cai, SP, Sasaki 1810.11000; Unal 1811.09151
- Higher orders: Adshead, Lozanov, Weiner 2105.01659; Garcia-Saenz, Pinol, Renaux-Petel, Werth, 2207.14267
- For exponential-tail of USR: Abe, Inui, Tada, Yokoyama, 2209.13891
- The impact of non-Gaussianity on induced GW is mild.

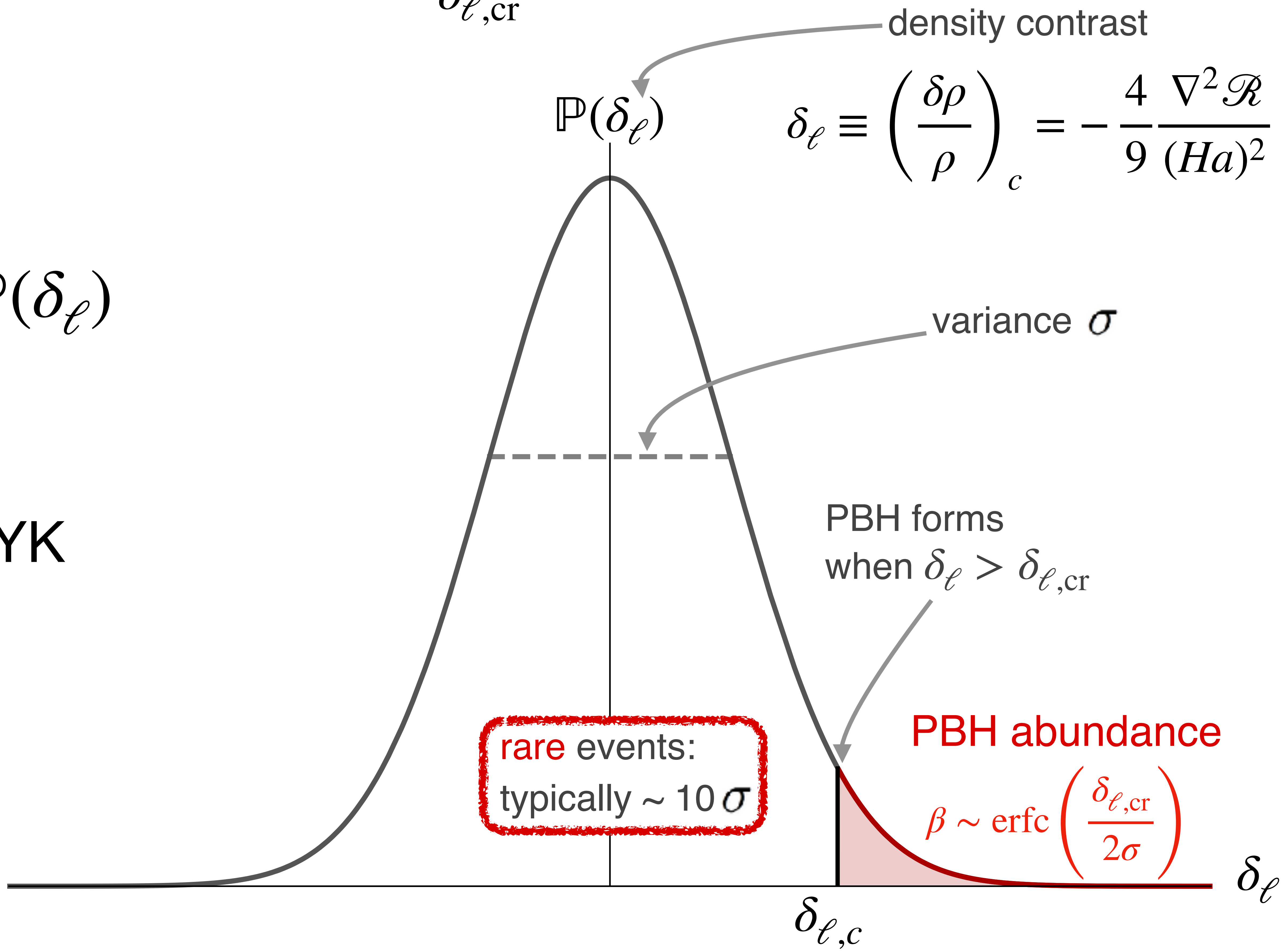




How to calculate: Press-Schechter

$$\left.\begin{array}{l} \mathscr{R} \xrightarrow{(1)} \delta_\ell \\ \mathbb{P}(\mathscr{R}) \xrightarrow{(2)} \mathbb{P}(\delta_\ell) \end{array}\right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \delta_{\ell,\mathrm{cr}}} \beta = \int_{\delta_{\ell,\mathrm{cr}}} \mathbb{P}(\delta_\ell) M(\delta_\ell) d\delta_\ell$$

- (1) Linear Poisson equation.
- (2) Gaussian PDF $\mathbb{P}(\mathscr{R})$ goes to Gaussian PDF $\mathbb{P}(\delta_\ell)$ by $\mathbb{P}(\mathscr{R})d\mathscr{R} = \mathbb{P}(\delta_\ell)d\delta_\ell$
- (3) Critical density contrast $\delta_{\ell,\mathrm{cr}}$ is given by the HYK limit (Harada, Yoo, Kohri, 1309.4201).
- (4) Window function matters for broad peaks.

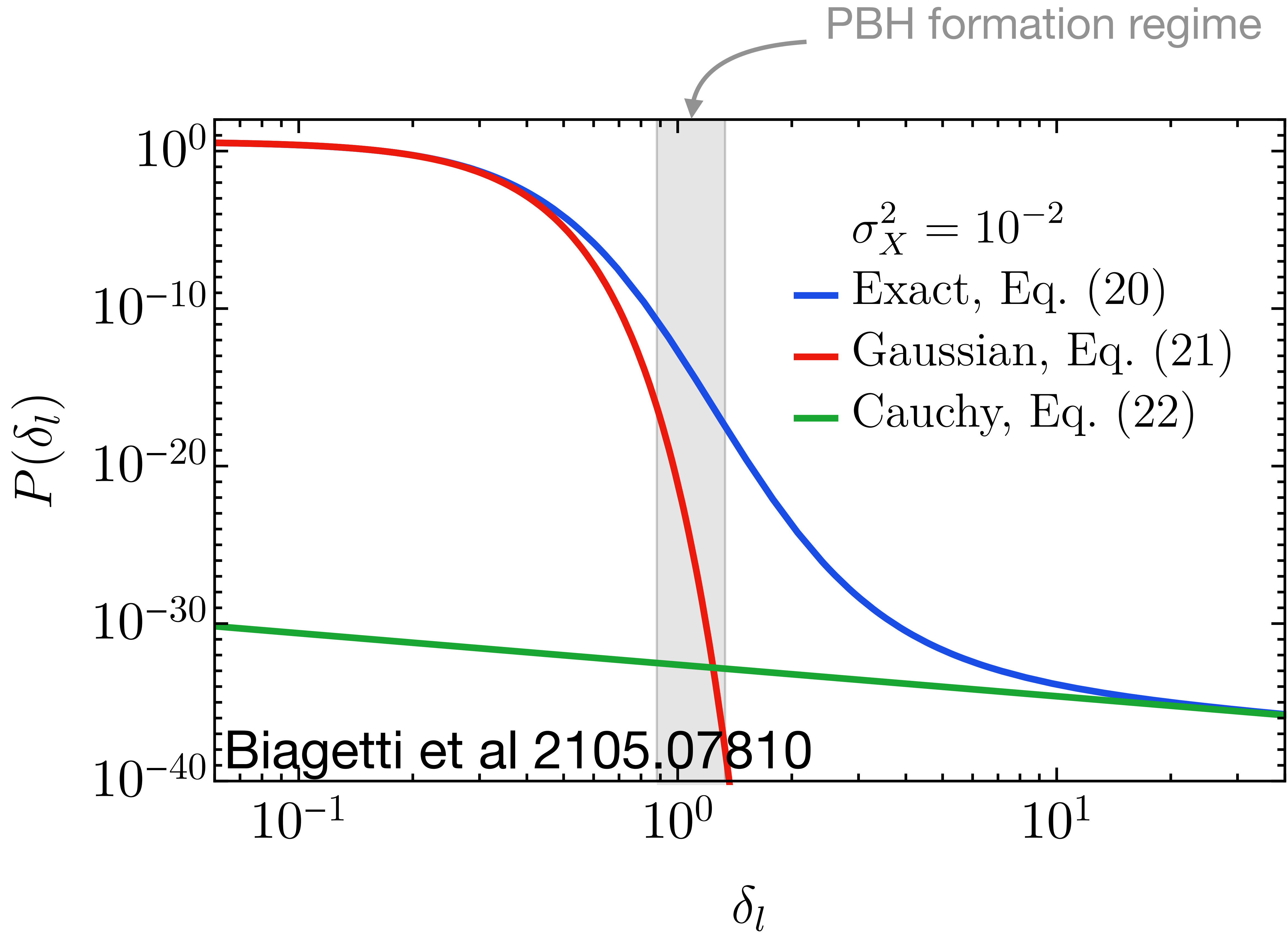


Why non-Gaussianity?

$$\left.\begin{array}{l}\mathscr{R} \xrightarrow{(1)} \mathscr{C} \\ \mathbb{P}(\mathscr{R}) \xrightarrow{(2)} \mathbb{P}(\mathscr{C})\end{array}\right\} \xrightarrow[\text{(4) Window function}]{\text{(3) given } \mathscr{C}_{\rm cr}} \beta = \int_{\mathscr{C}_{\rm cr}} \mathbb{P}(\mathscr{C}) M(\mathscr{C}) d\mathscr{C}$$

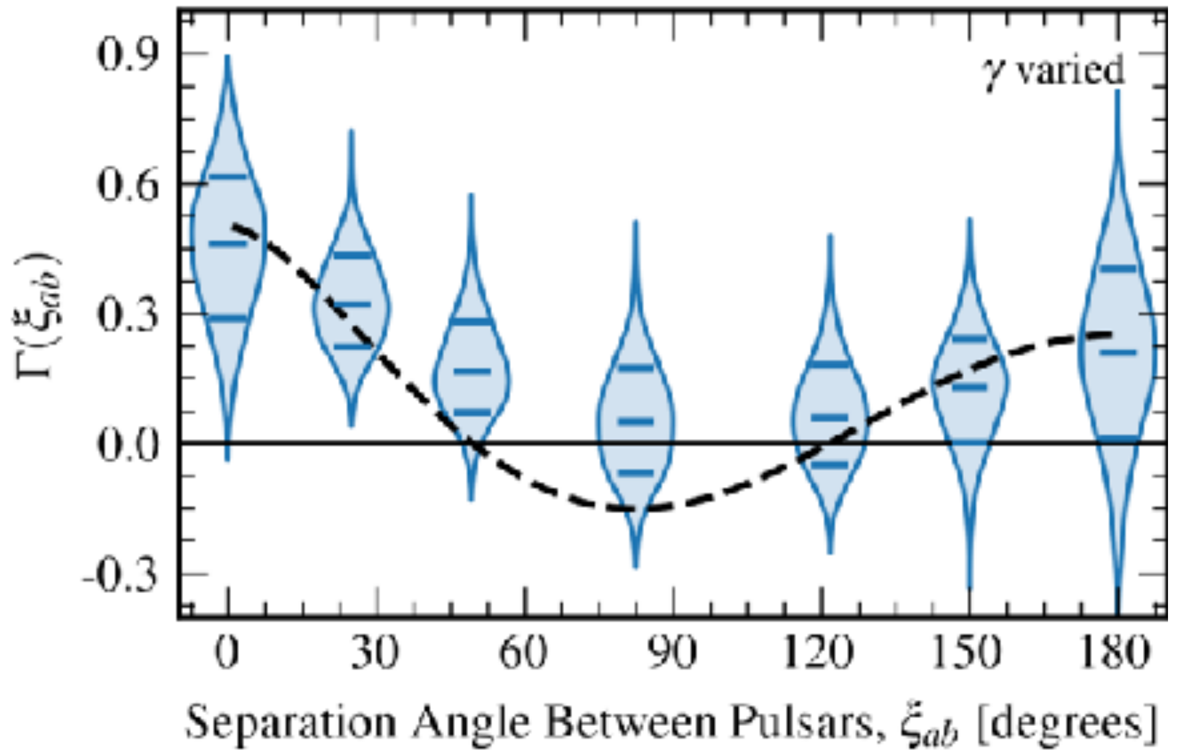
Non-Gaussianity enters in different processes

- (1) Use the compaction $\mathscr{C}$ function to calculate, which is connected to $\mathscr{R}$ by nonlinear Poisson equation. (Harada et al 1503.03934; De Luca et al 1904.00970.)
- (2) PDF $\mathbb{P}(\mathscr{R})$ could be non-Gaussian, which goes to non-Gaussian PDF $\mathbb{P}(\mathscr{C})$. (Main topic of this talk)
- (3) Critical density contrast $\mathscr{C}_{\rm cr}$ given by numerical simulations. (Musco 1809.02127; Escrivà et al 1907.13311)

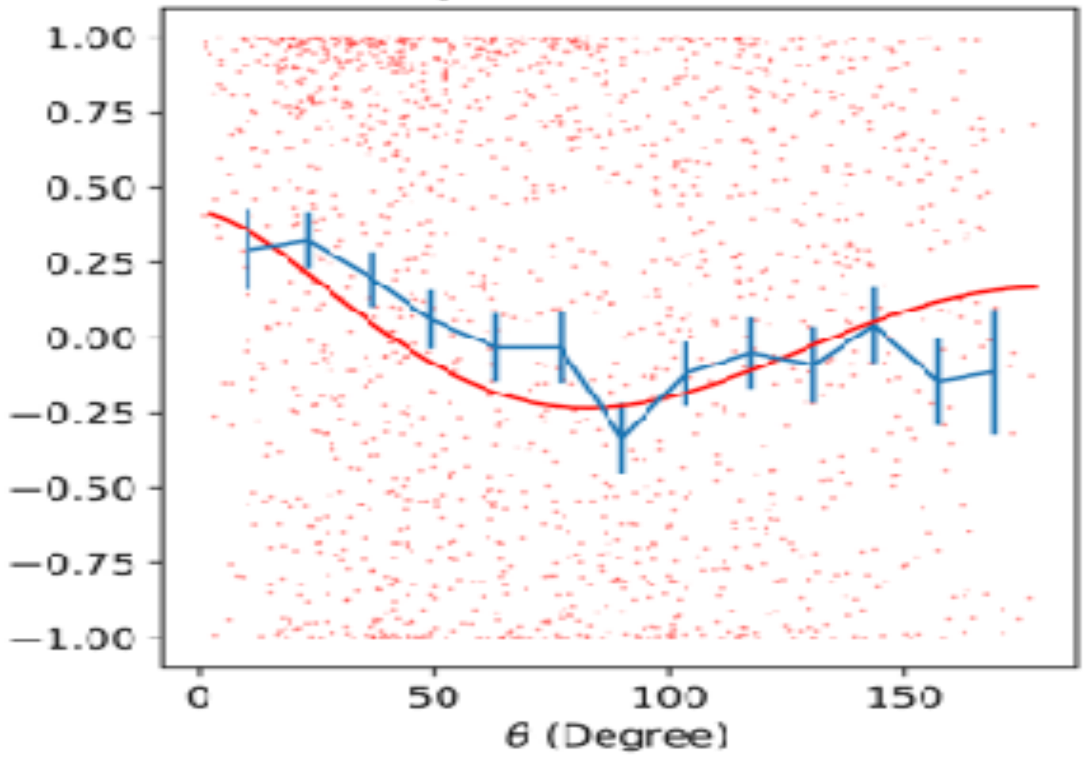


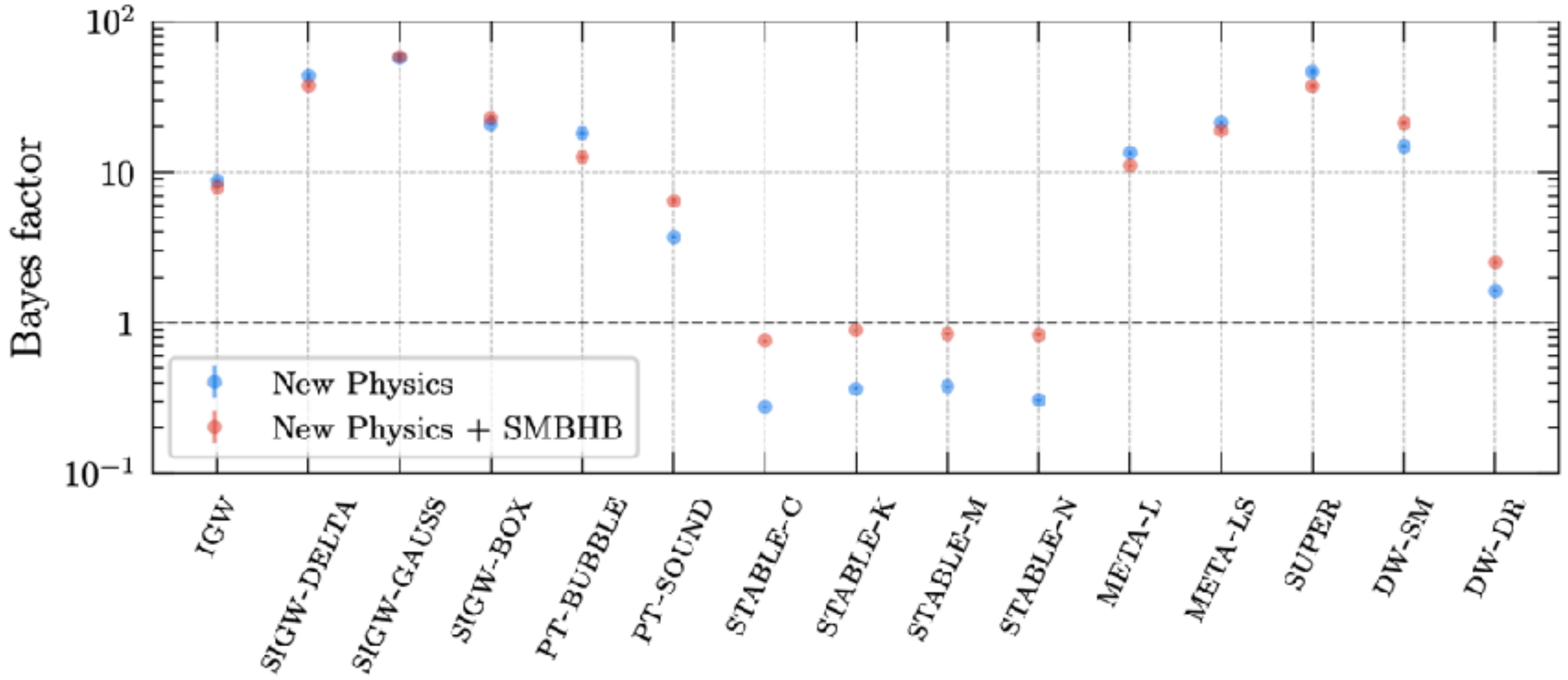
Application: nHz SGWB

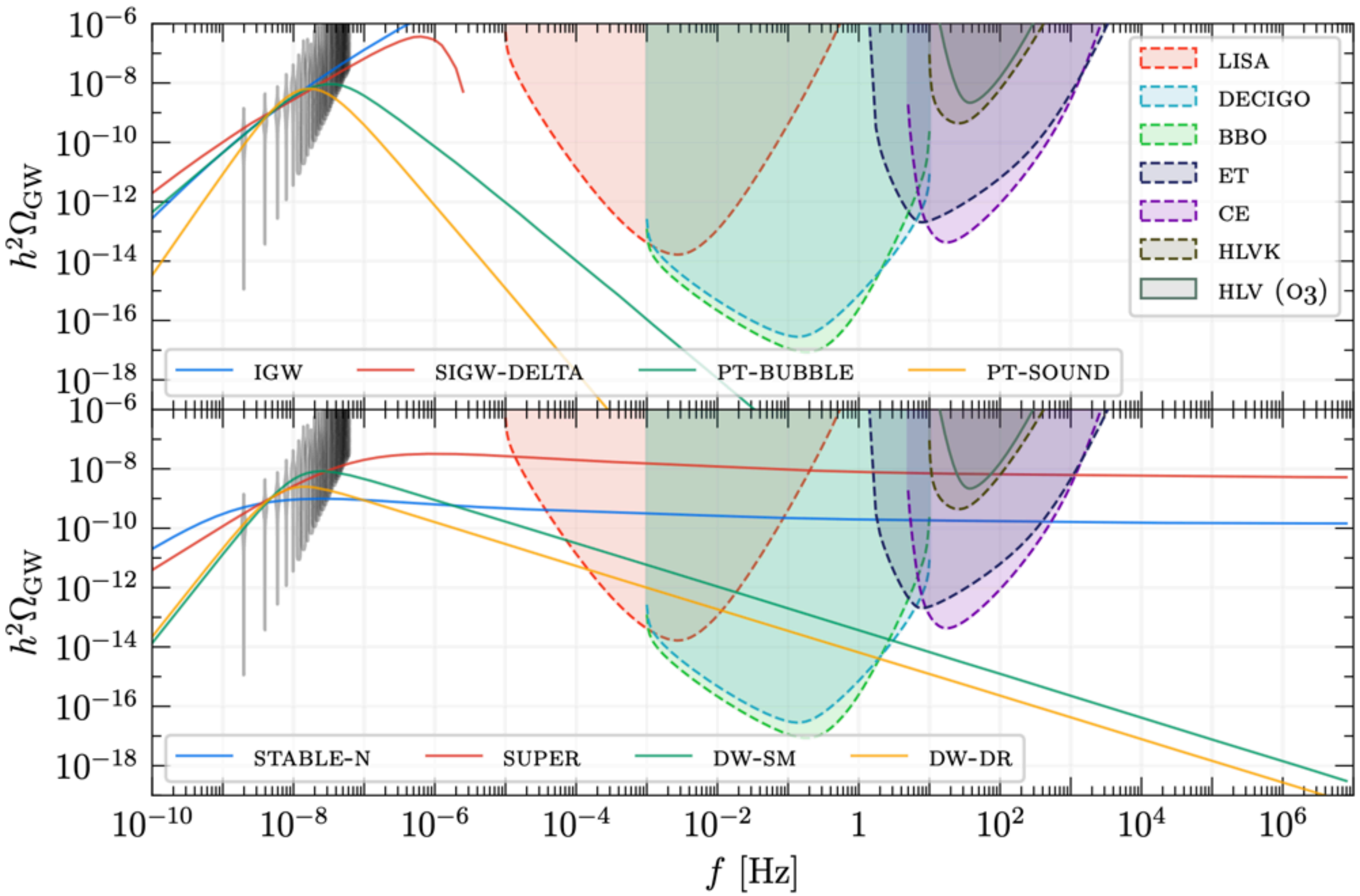
NANOGrav, 2306.16213



CPTA, 2306.16216







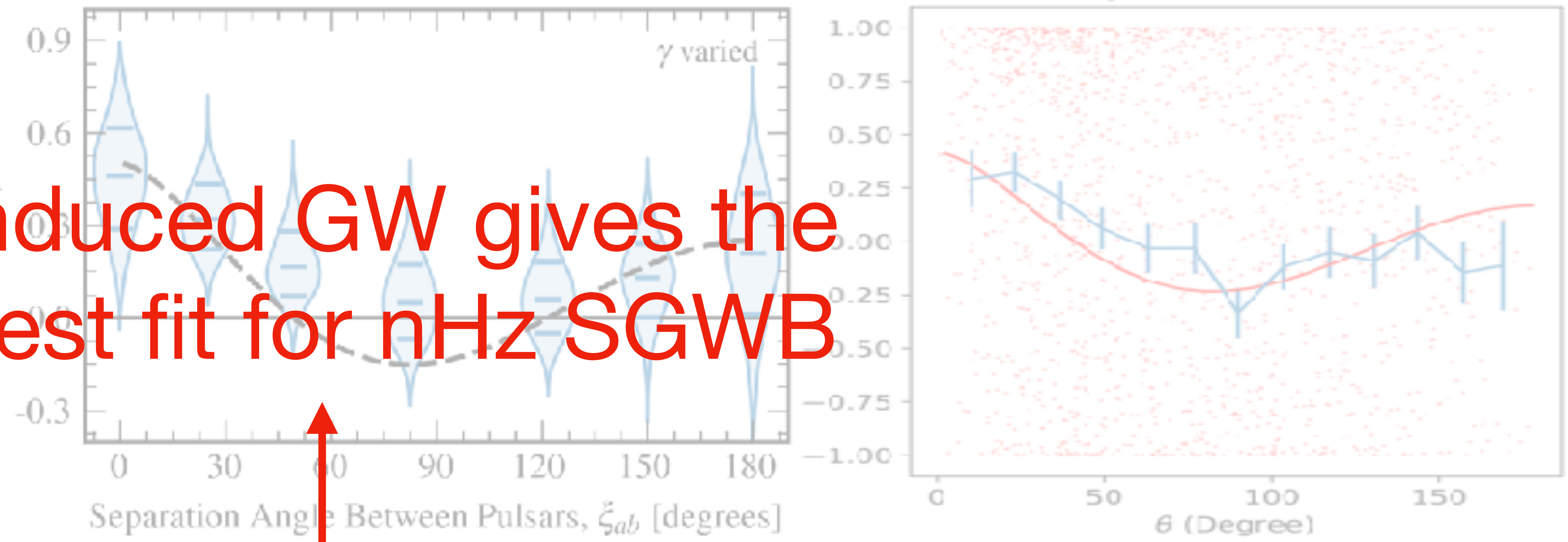
NANOGrav, 2306.16219

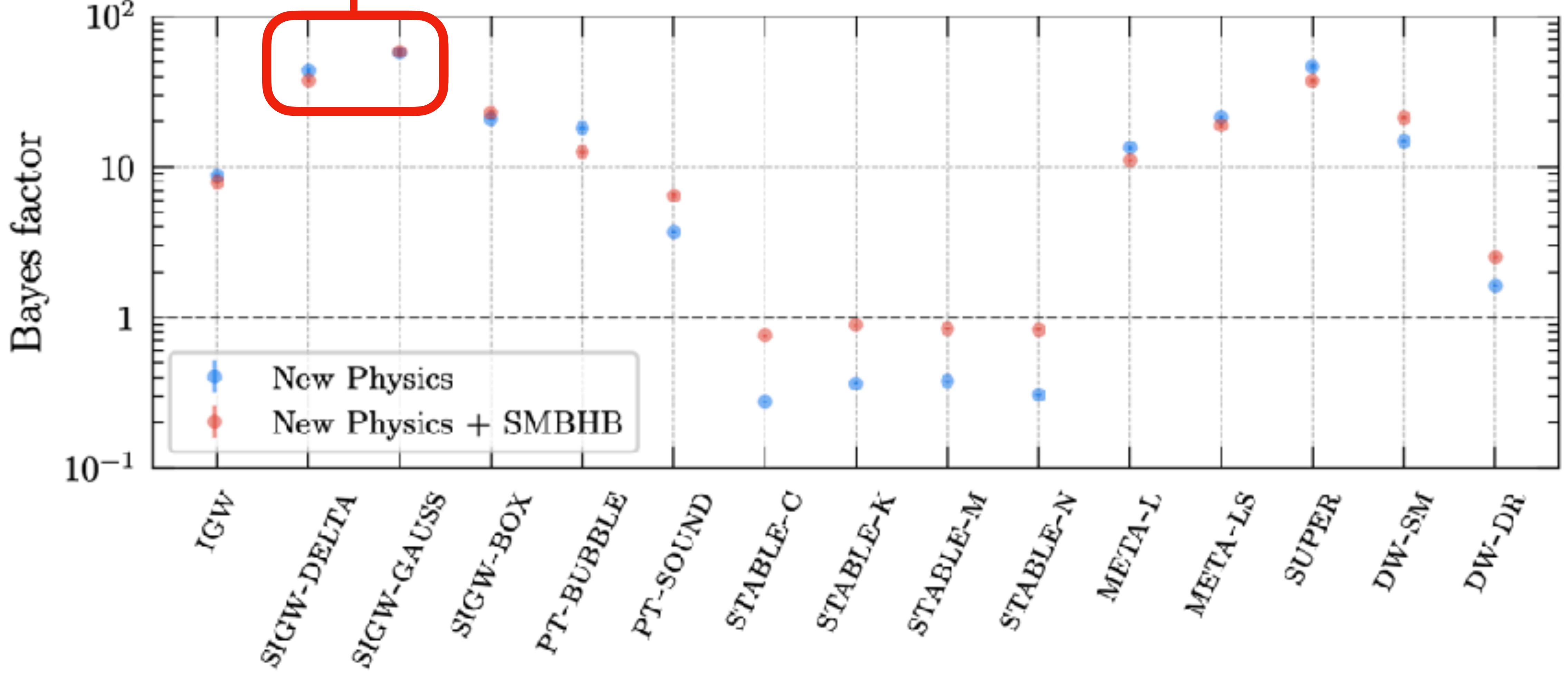
Application: nHz SGWB

NANOGrav, 2306.16213

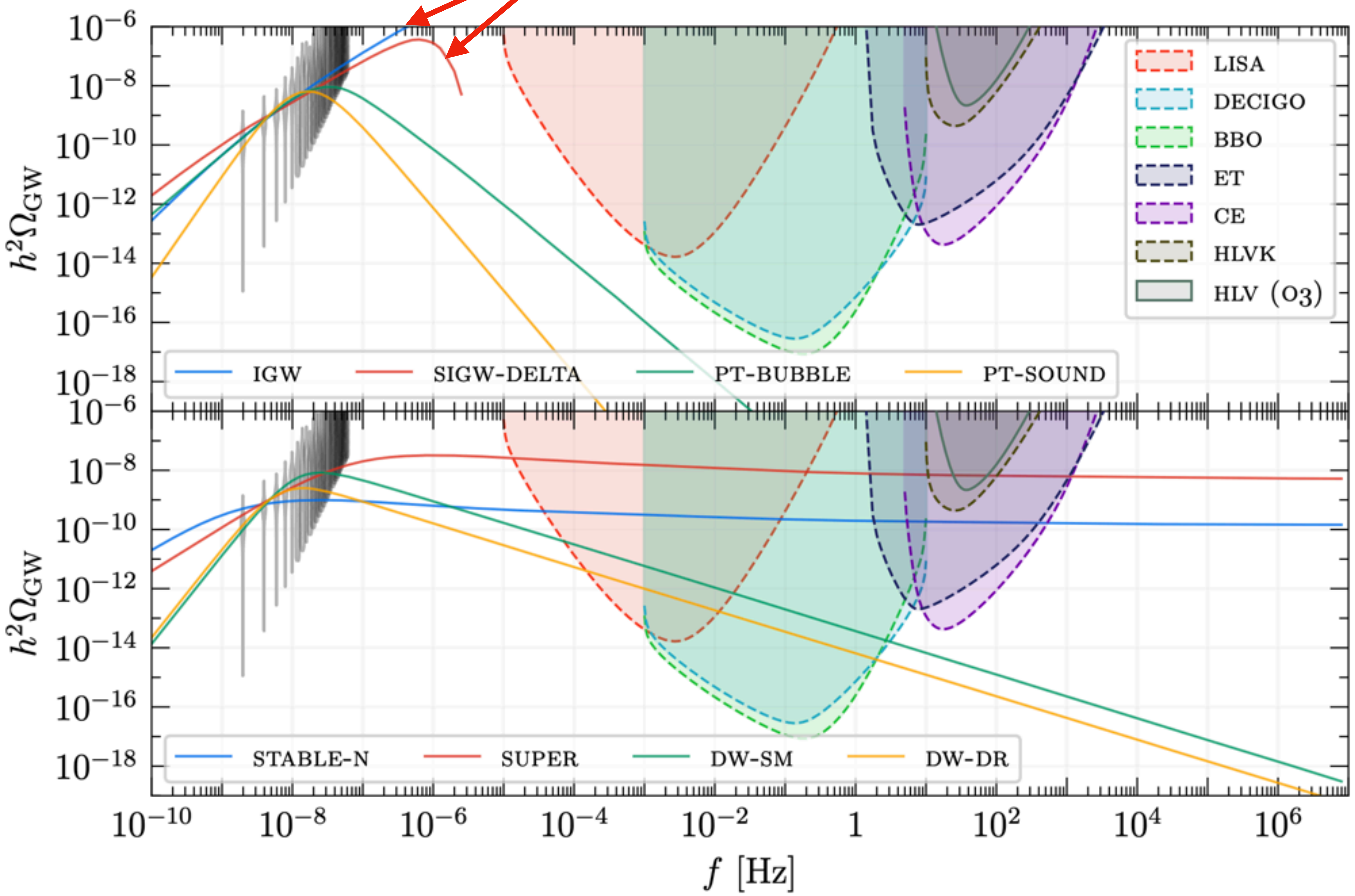
CPTA, 2306.16216

Induced GW gives the best fit for nHz SGWB



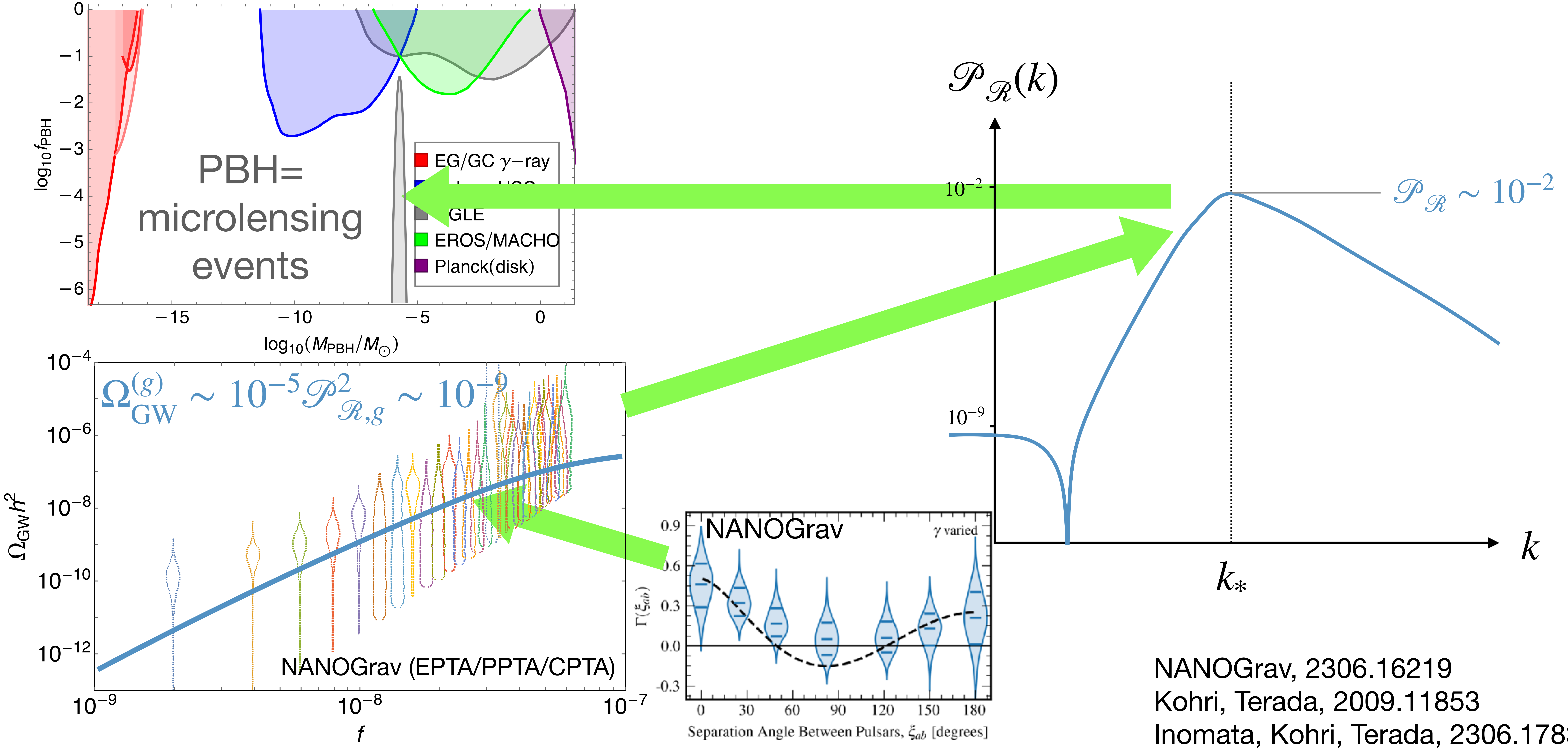


Induced GW



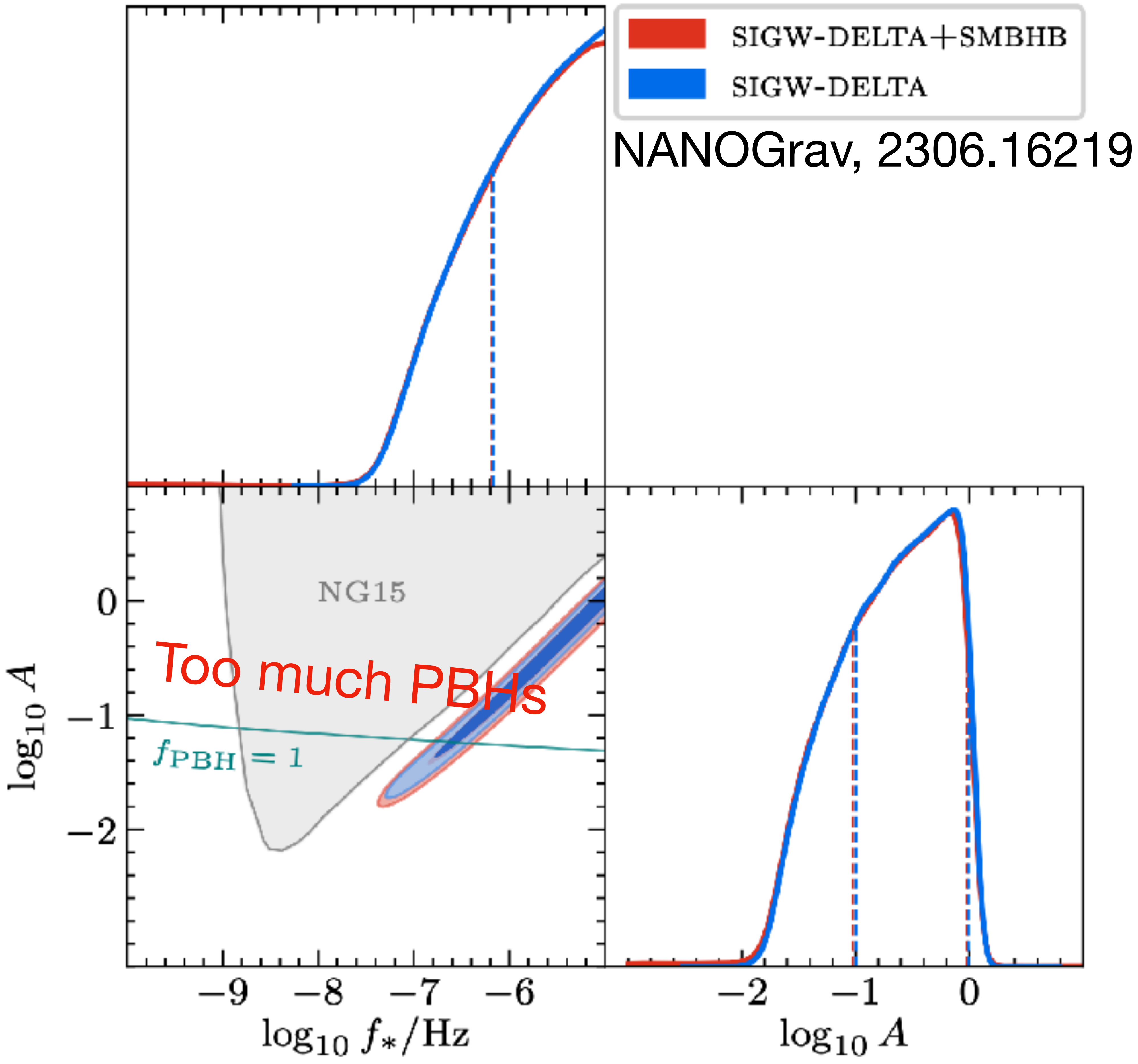
NANOGrav, 2306.16219

Crosscheck by PBH and IGW



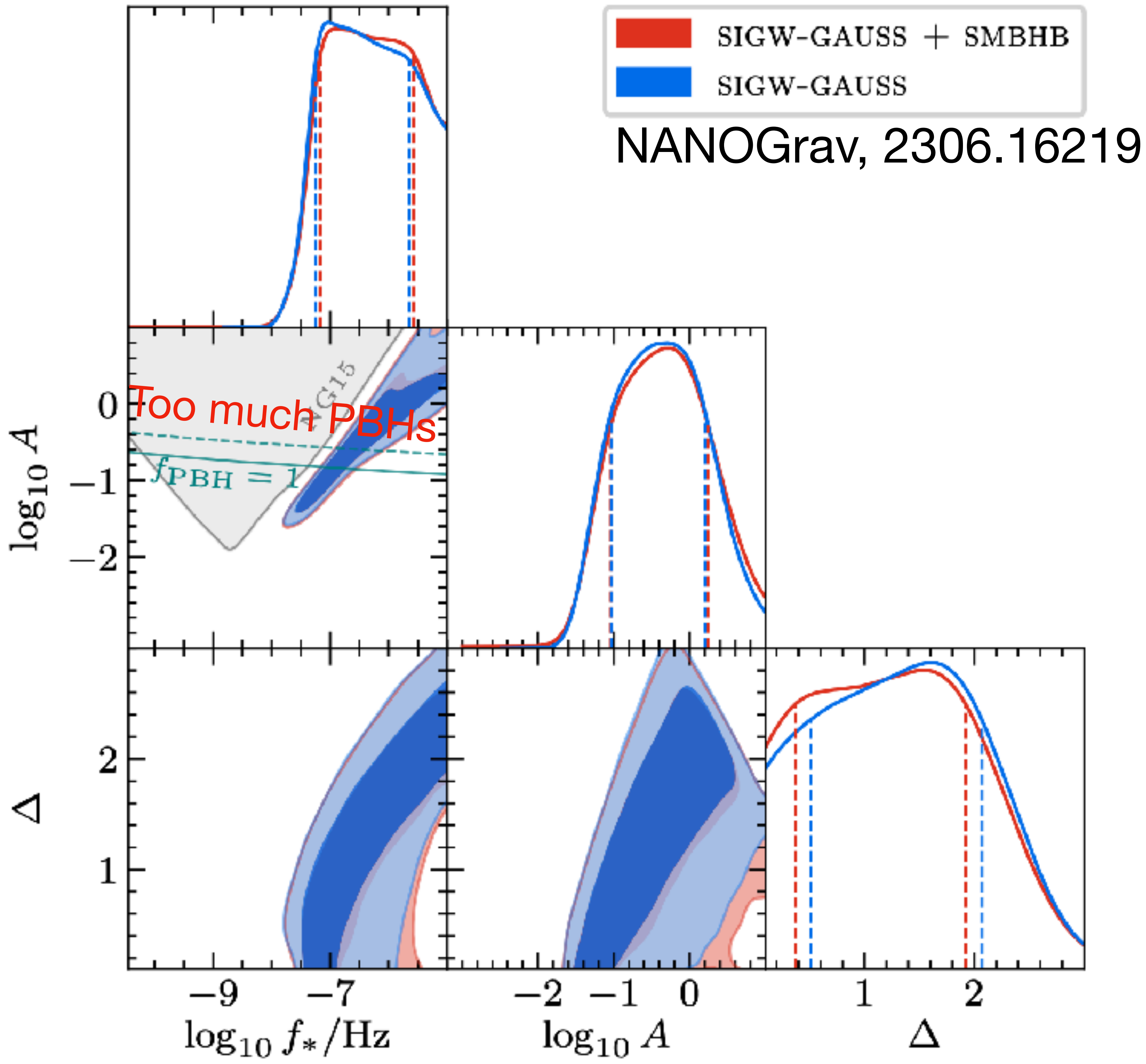
NANOGrav, 2306.16219
Kohri, Terada, 2009.11853
Inomata, Kohri, Terada, 2306.17834

IGW as nHz SGWB



$$\mathscr{P}_{\mathscr{R}} = A\delta(\ln k - \ln k_*)$$

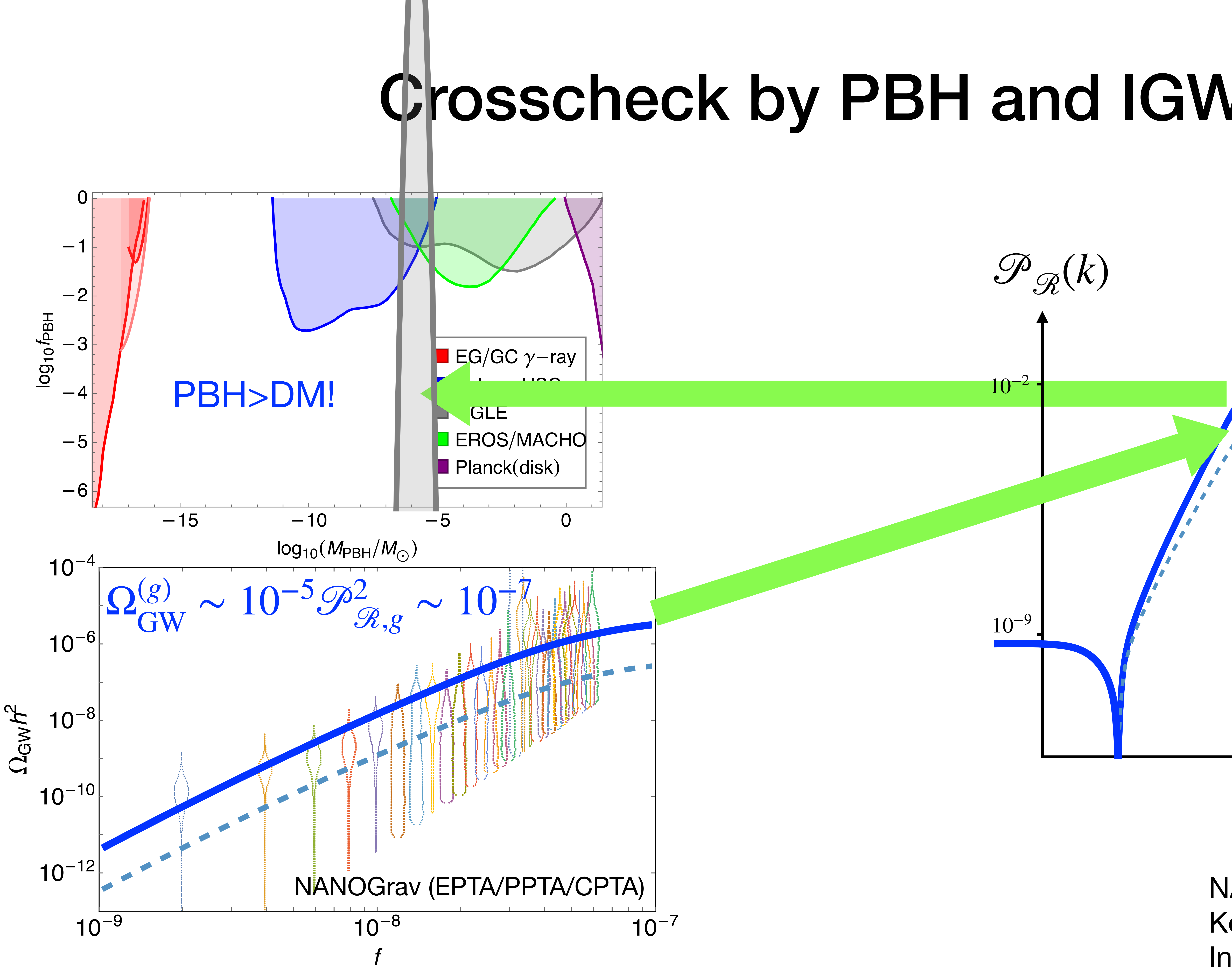
monochromatic

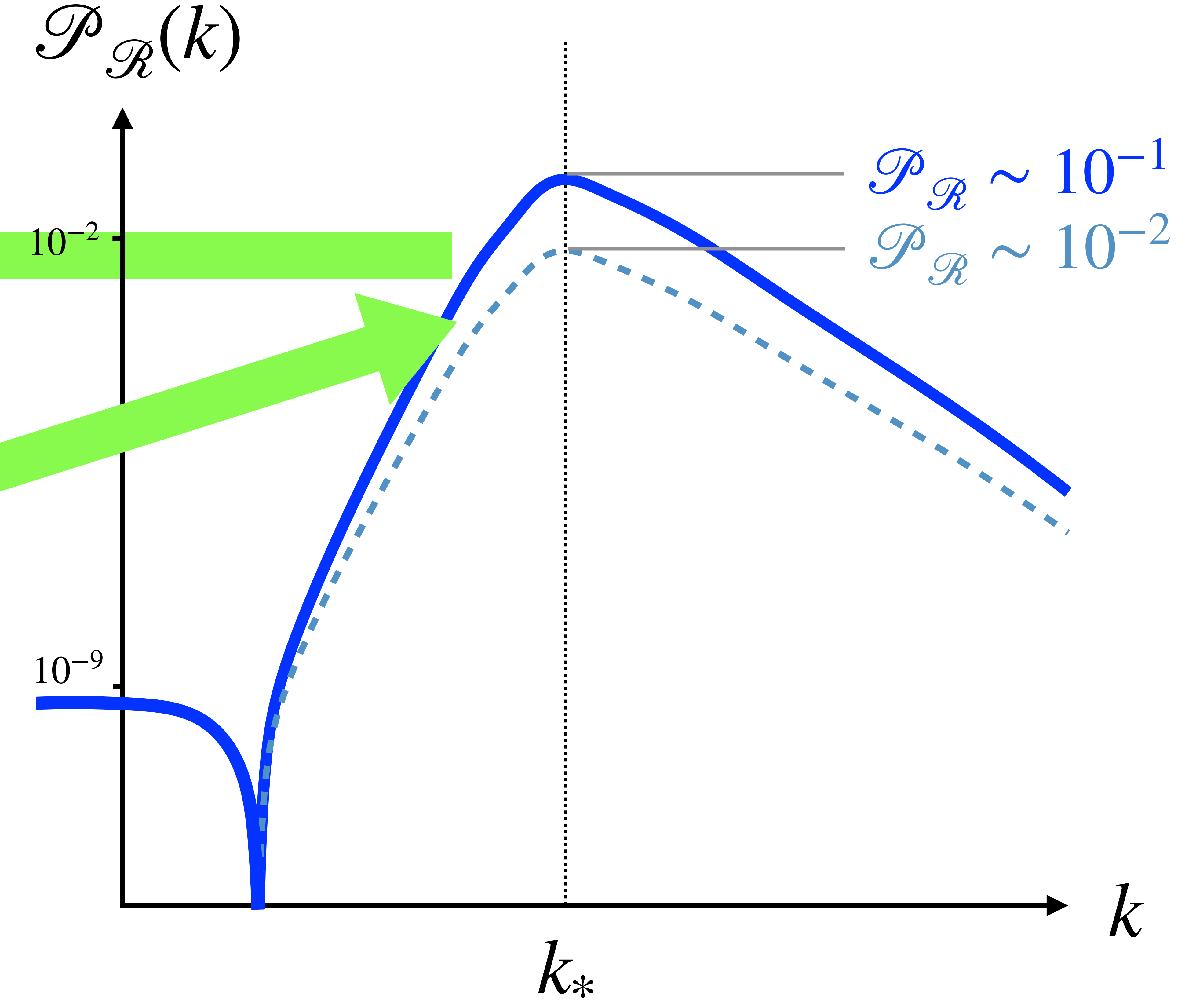


$$\mathscr{P}_{\mathscr{R}} = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{(\ln k - \ln k_*)^2}{2\Delta^2}\right)$$

lognormal [SP and Sasaki 2005.12306]

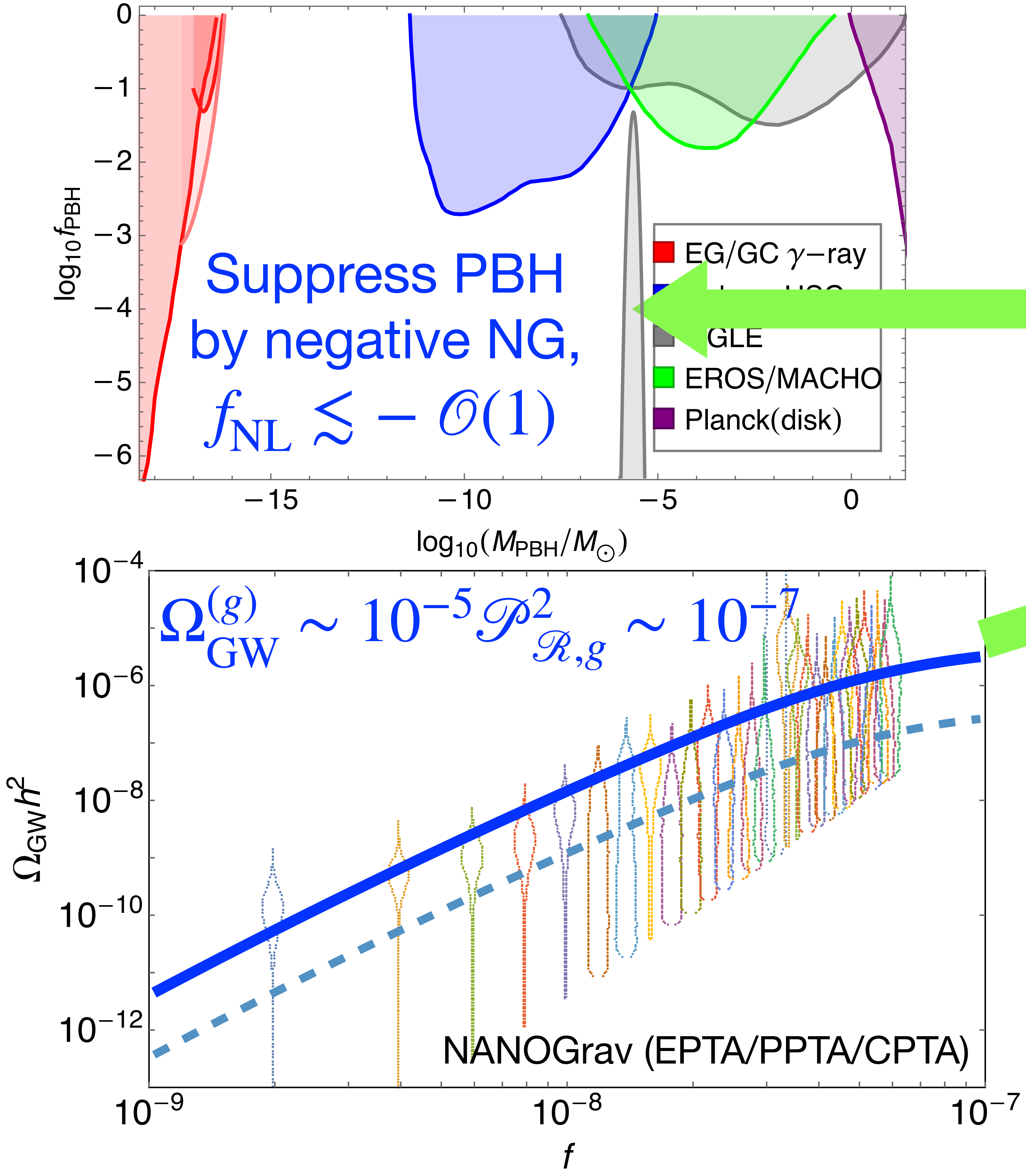
Crosscheck by PBH and IGW

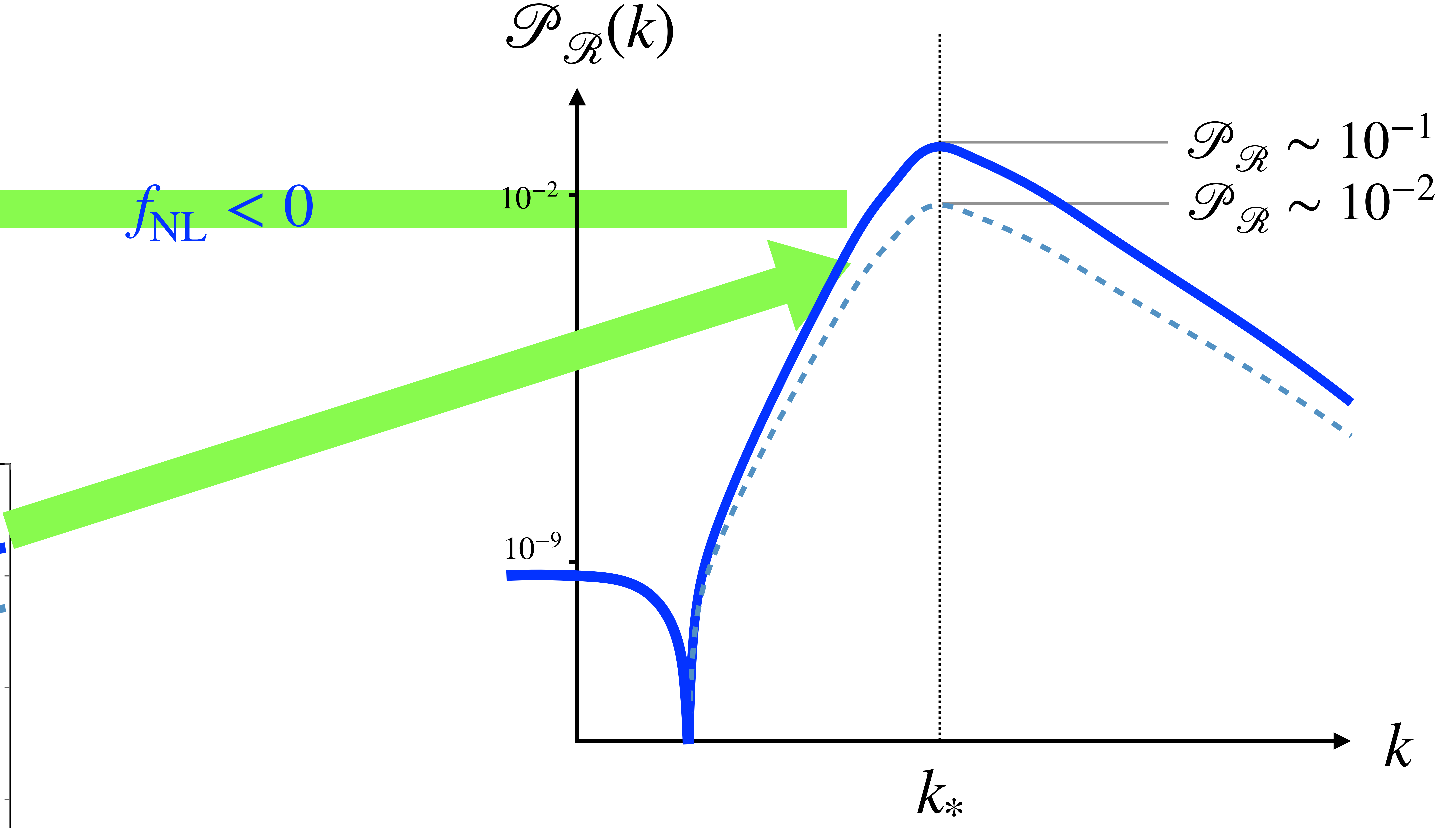




NANOGrav, 2306.16219
Kohri, Terada, 2009.11853
Inomata, Kohri, Terada, 2306.17834

Crosscheck by PBH and IGW

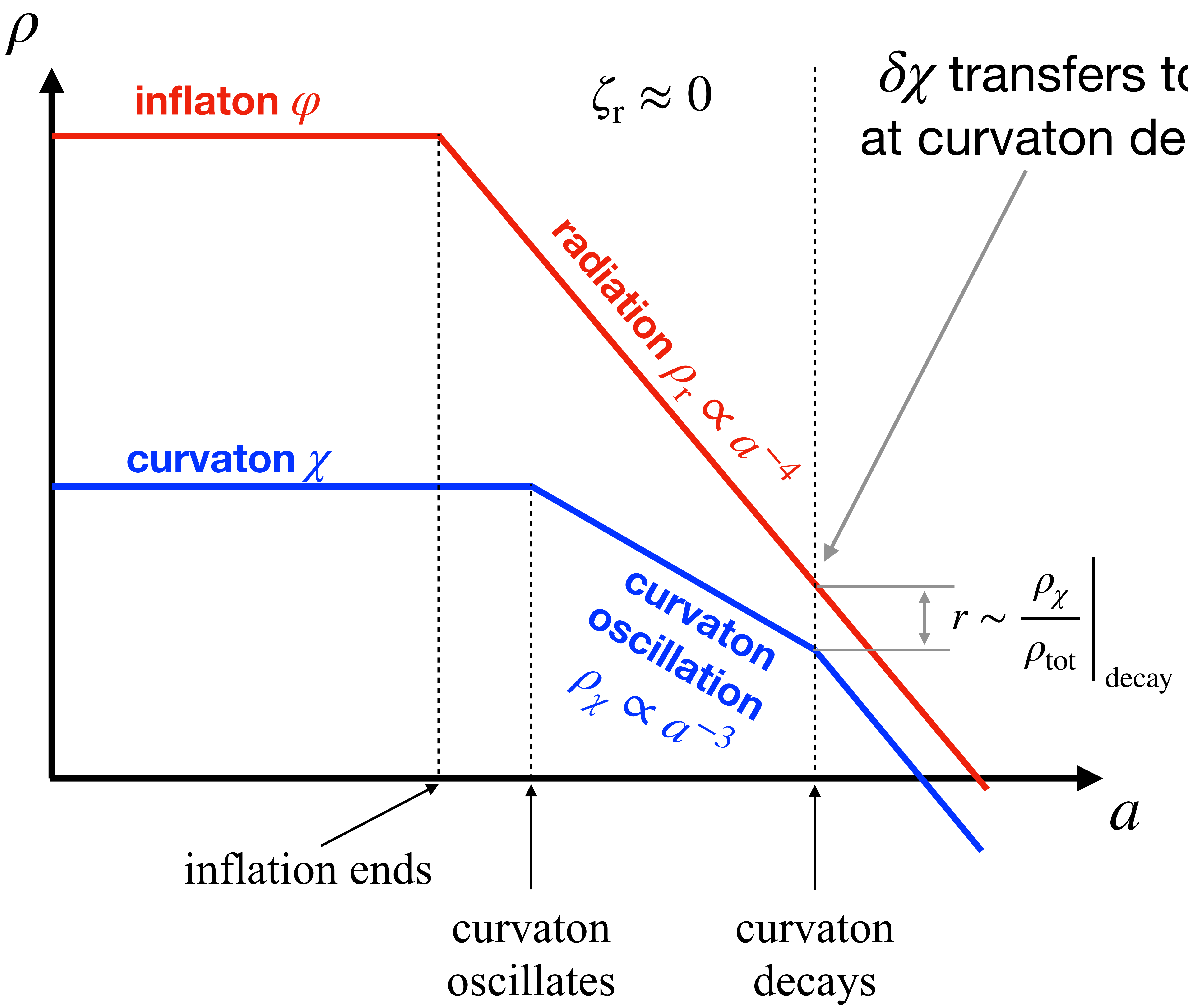




Franciolini et al, 2306.17149
Liu et al, 2307.01102

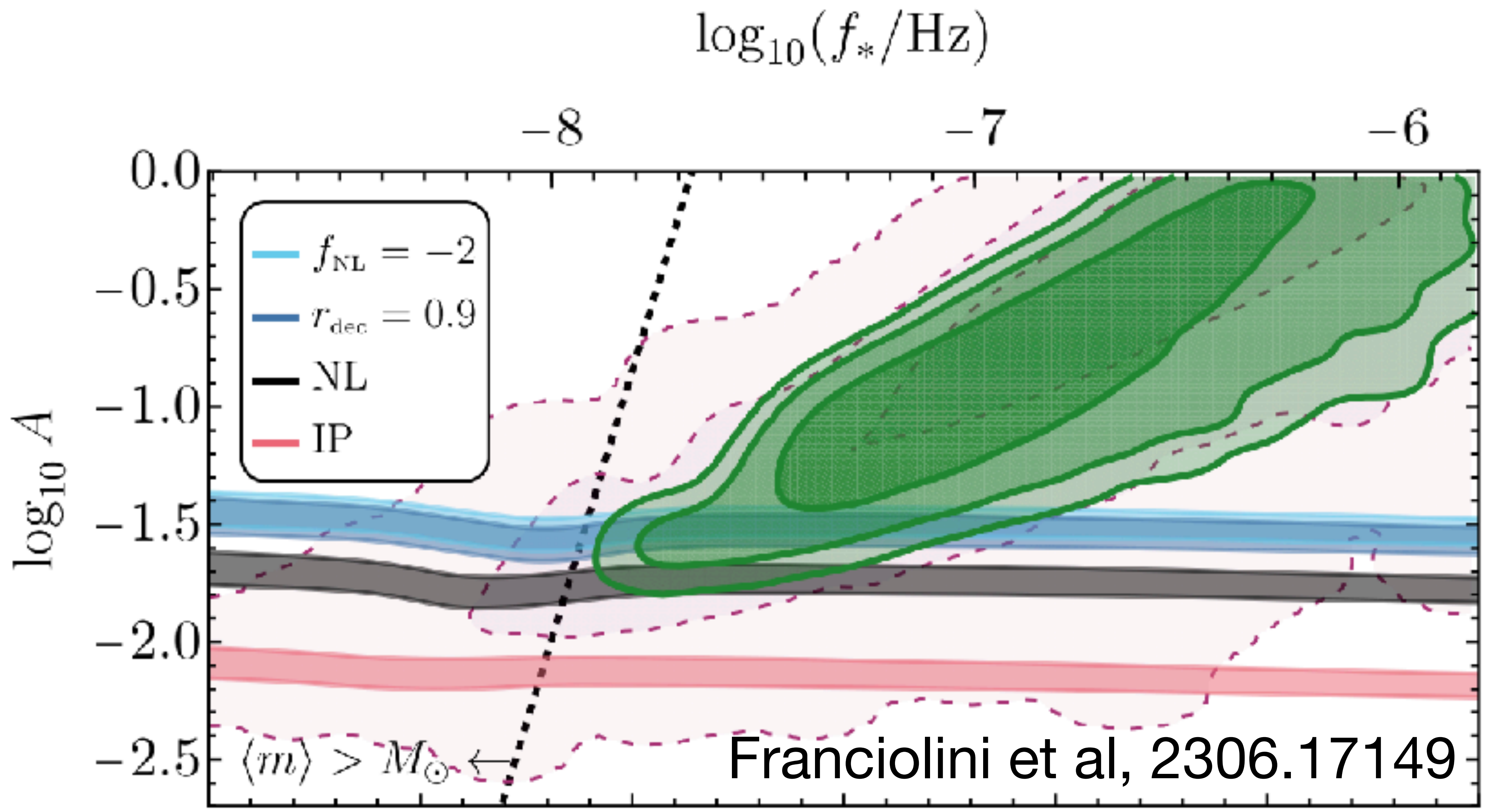
IGW as nHz SGWB

SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728



$$\zeta = \zeta(\delta\chi/\chi) \longrightarrow \begin{cases} \frac{r}{3}\left[2\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2\right] & \text{when } r \ll 1 \\ \frac{2}{3}\ln\left|1 + \frac{\delta\chi}{\chi}\right| & \text{when } r \sim 1 \end{cases}$$

- $\zeta(\delta\chi)$ degenerates to a logarithmic relation ($f_{\rm NL} = -5/4$) when the curvaton dominates.



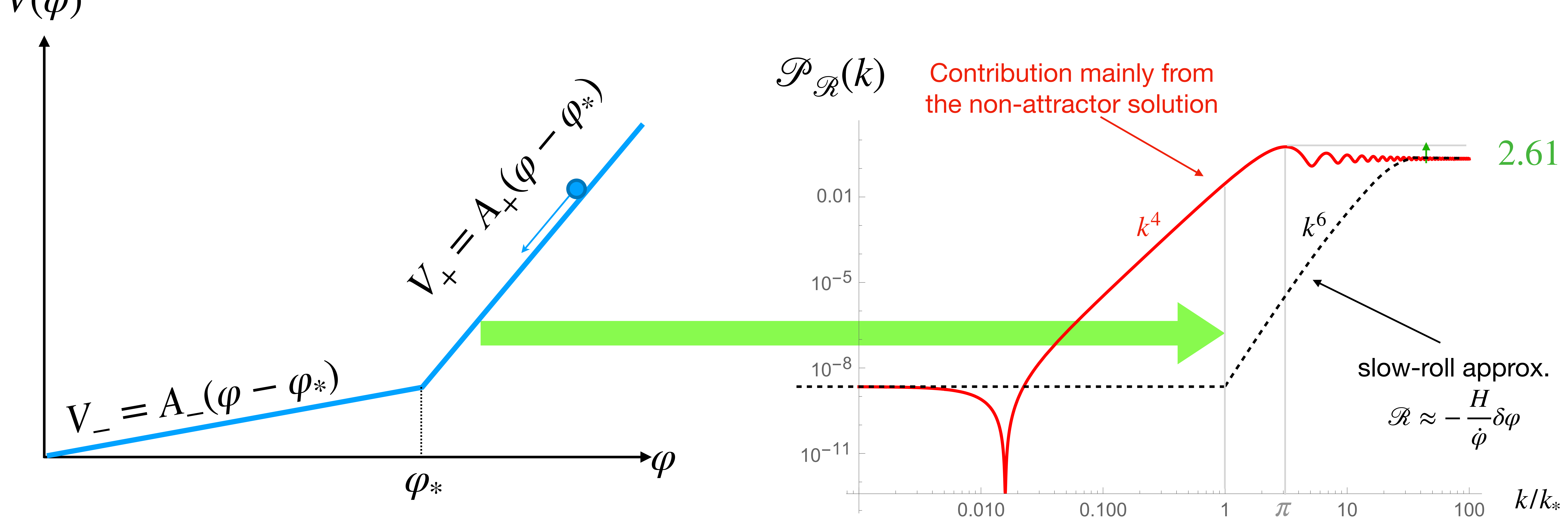
Franciolini et al, 2306.17149

Nonlinearity of the curvature perturbation

A general relation between $\mathscr{R}$ and $\delta\varphi$

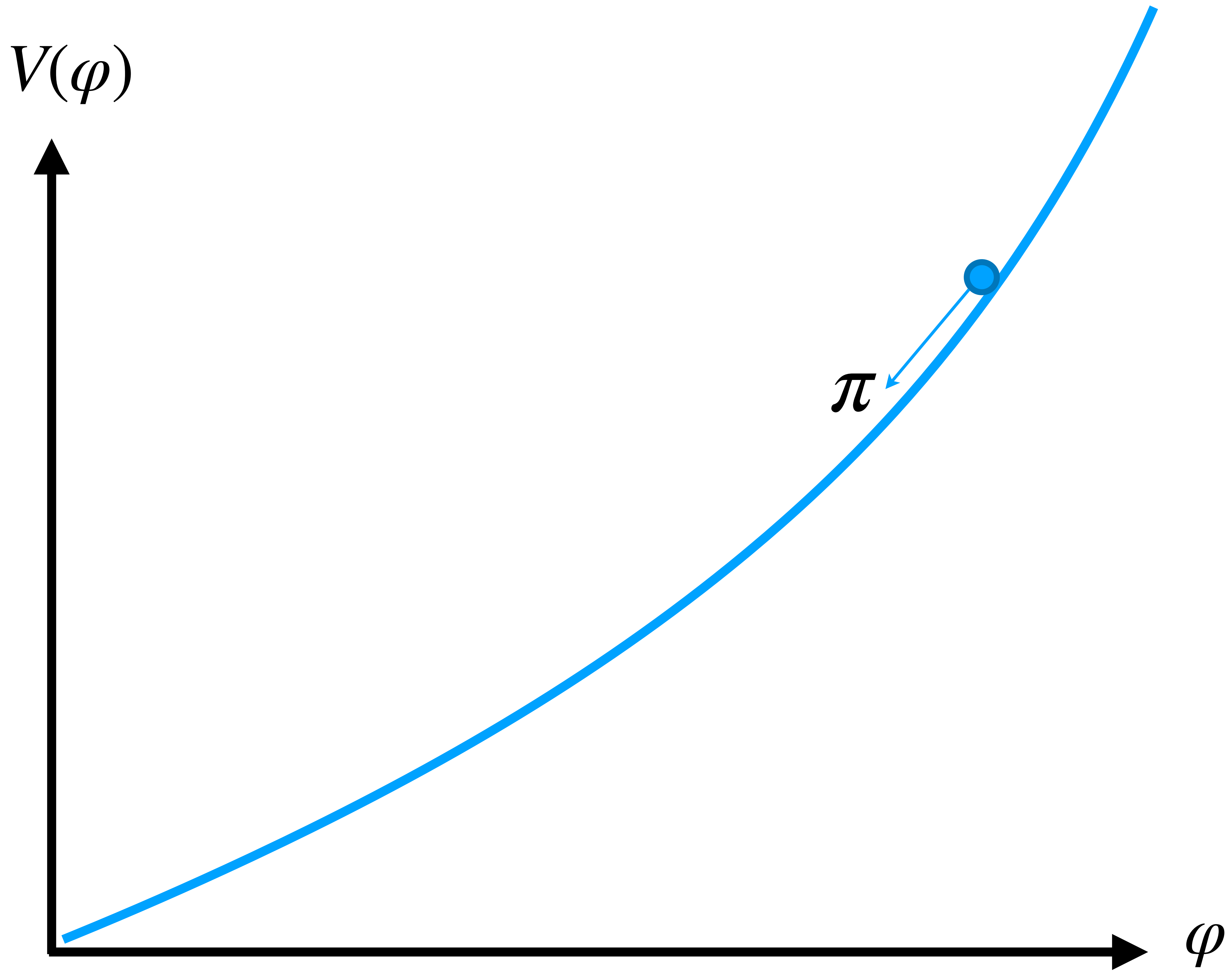
Ultra-slow-roll Inflation

Starobinsky's linear potential model



Starobinski, JETP Lett. 55, 489
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP, Jianing Wang, 2209.14183
Domenech, Vargas, Vargas, 2309.05750

Gaussian Curvature Perturbation



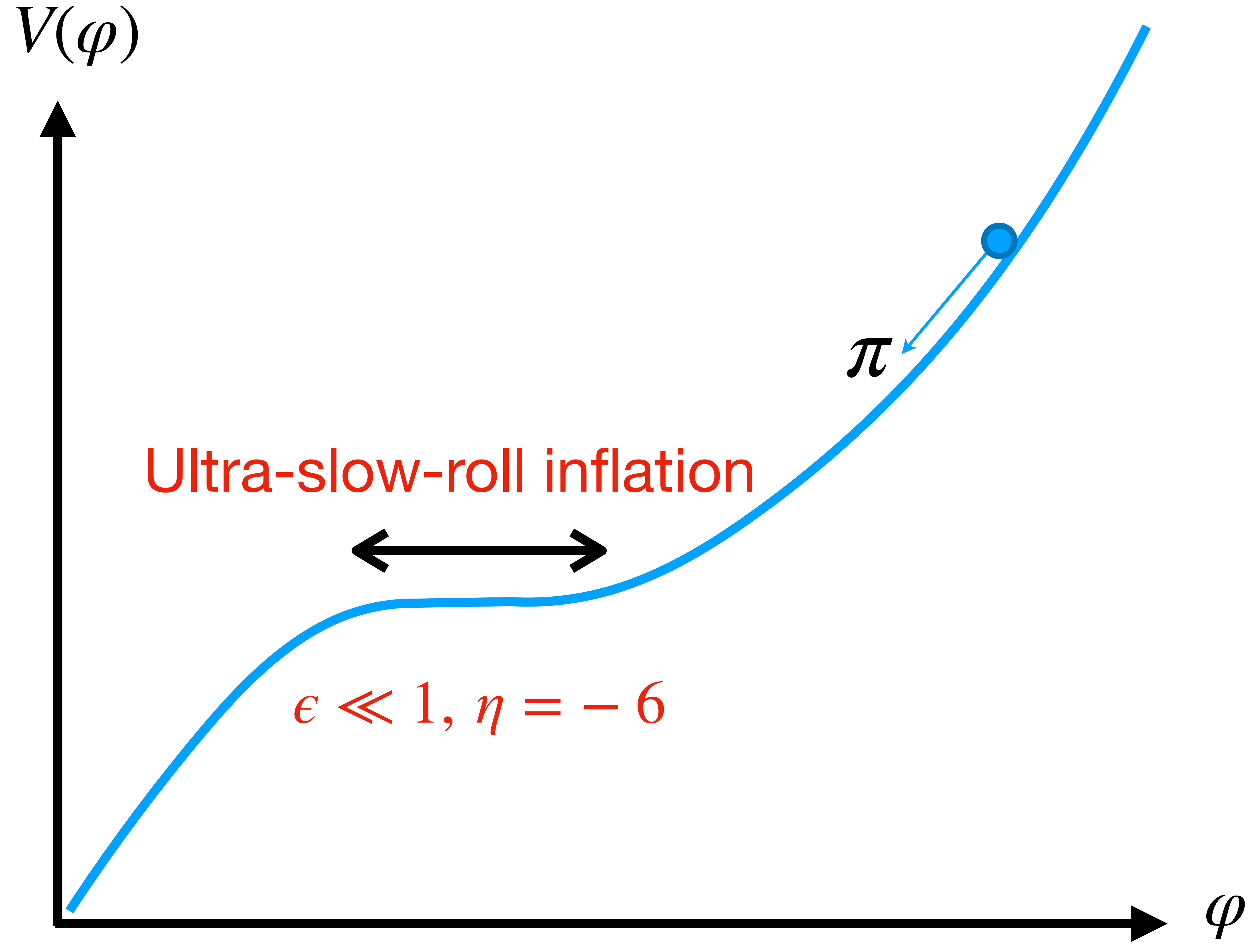
$$\mathcal{R} = \delta N \approx N_{,\varphi}\delta\varphi + \frac{1}{2}N_{,\varphi\varphi}\delta\varphi^2 + \cdots$$

$$= -H\frac{\delta\varphi}{\dot{\varphi}} + \frac{3}{5}\boxed{f_{\rm NL}}\left(-H\frac{\delta\varphi}{\dot{\varphi}}\right)^2\cdots$$

$f_{\rm NL} \rightarrow \mathcal{O}(\epsilon, \eta)$

Stewart and Sasaki, astro-ph/9507001
Lyth and Roquigez, astro-ph/0504045
Maldacena, astro-ph/0210603

Logarithmic Relation in the USR inflation



$\mathcal{O}(\epsilon, \eta) \sim \mathcal{O}(1)$

$$\mathcal{R} = \delta N = N_{,\varphi}\delta\varphi + \frac{1}{2}N_{,\varphi\varphi}\delta\varphi^2 + \cdots$$

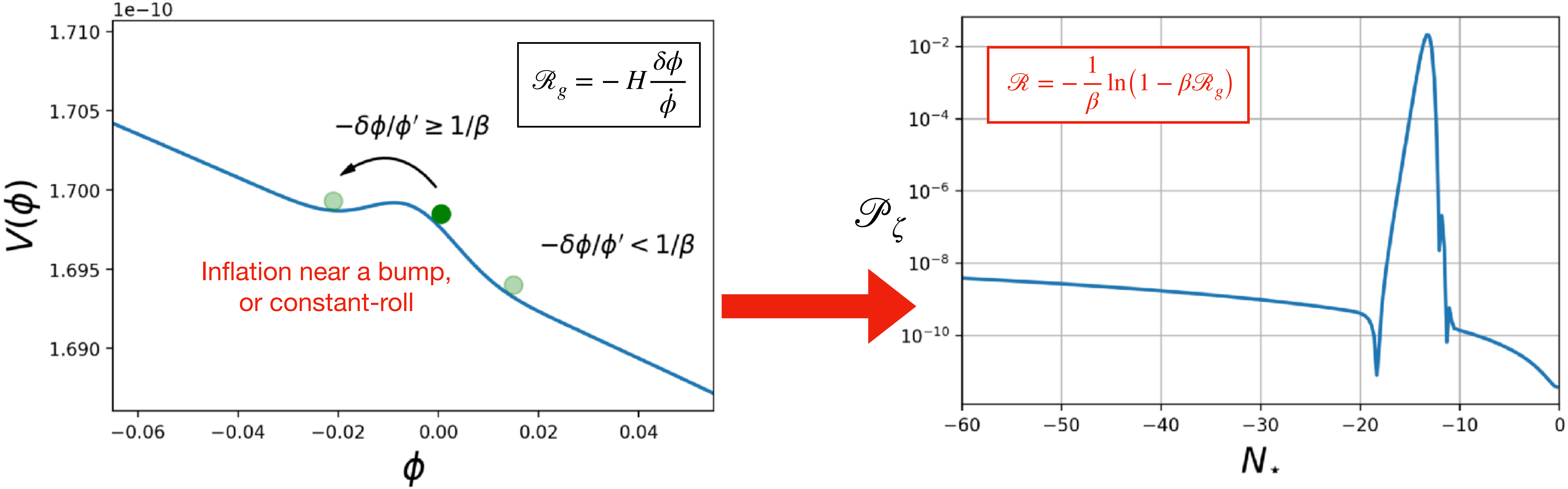
$$+N_{,\pi}\delta\pi + \frac{1}{2}N_{,\pi\pi}\delta\pi^2 + \cdots$$

(For USR) $$= -\frac{1}{3}\ln\left(1 + \frac{3\delta\phi}{\pi_*}\right).$$

$$\left(f_{\rm NL} = \frac{5}{2}, \quad g_{\rm NL} = -\frac{25}{3}, \quad \cdots\right)$$

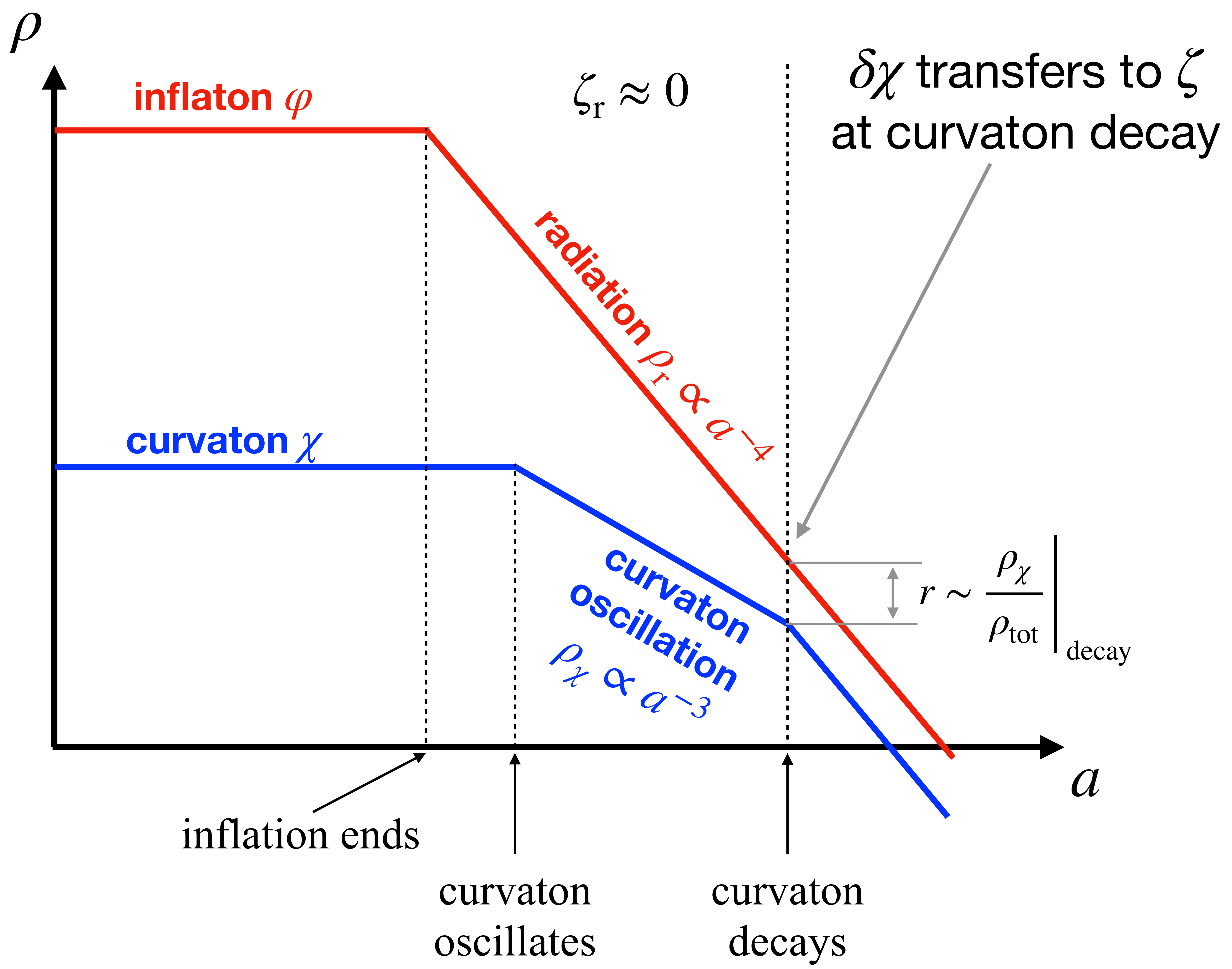
Namjoo, Firouzjahi, Sasaki, 1210.3692
Chen, Firouzjahi, Komatsu, Namjoo, Sasaki, 1308.5341
Cai, Chen, Namjoo, Sasaki, Wang, Wang, 1712.09998
Biagetti, Franciolini, Kehagias, Riotto, 1804.07124
Passaglia, Hu, Motohashi, 1812.08243
Also verified by stochastic approach, see e.g.
Pattison et al 2101.05741

Logarithmic Relation in Constant-Roll Inflation



Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990

Curvaton Scenario



$$\Longrightarrow \quad e^{4\zeta} - \frac{4r}{3+r}\left(1+\frac{\delta\chi}{\chi}\right)^2 e^{\zeta} + \frac{3r-3}{3+r} = 0$$

$$\zeta = \zeta(\delta\chi/\chi) \longrightarrow \begin{cases} \frac{r}{3}\left[2\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2\right] & \text{when } r \ll 1 \\ \frac{2}{3}\ln\left|1+\frac{\delta\chi}{\chi}\right| & \text{when } r \sim 1 \end{cases}$$

- $\zeta(\delta\chi)$ is strictly quadratic when the curvaton is negligible, $f_{NL} = 5/(4r) \gg 1$.
- $\zeta(\delta\chi)$ degenerates to a logarithmic relation ($f_{NL} = -5/4$) when the curvaton dominates.

SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728

$$\mathcal{R}(\delta\varphi)$$

$$\mathcal{R} = \frac{1}{\lambda}\ln\left(1 + \lambda\mathcal{R}_g\right)$$

$\lambda \ll 1$

$\left(f_{NL} = -\frac{5}{6}\lambda\right)$

$$\mathcal{R} = -H\frac{\delta\varphi}{\dot\varphi} + \frac{3}{5}f_{\rm NL}\left(-H\frac{\delta\varphi}{\dot\varphi}\right)^2$$

Stewart and Sasaki, 1995
Lyth and Roquigez, 2005

$\lambda \to 0$ (?)

$\lambda = -3$

$$\mathcal{R} = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right)$$

Cai et al 1712.09998
Biagetti et al 1804.07124
Passaglia et al 1812.08243

$\lambda = 3/2$

$$\mathcal{R} = \frac{2}{3}\ln(1+\delta)$$

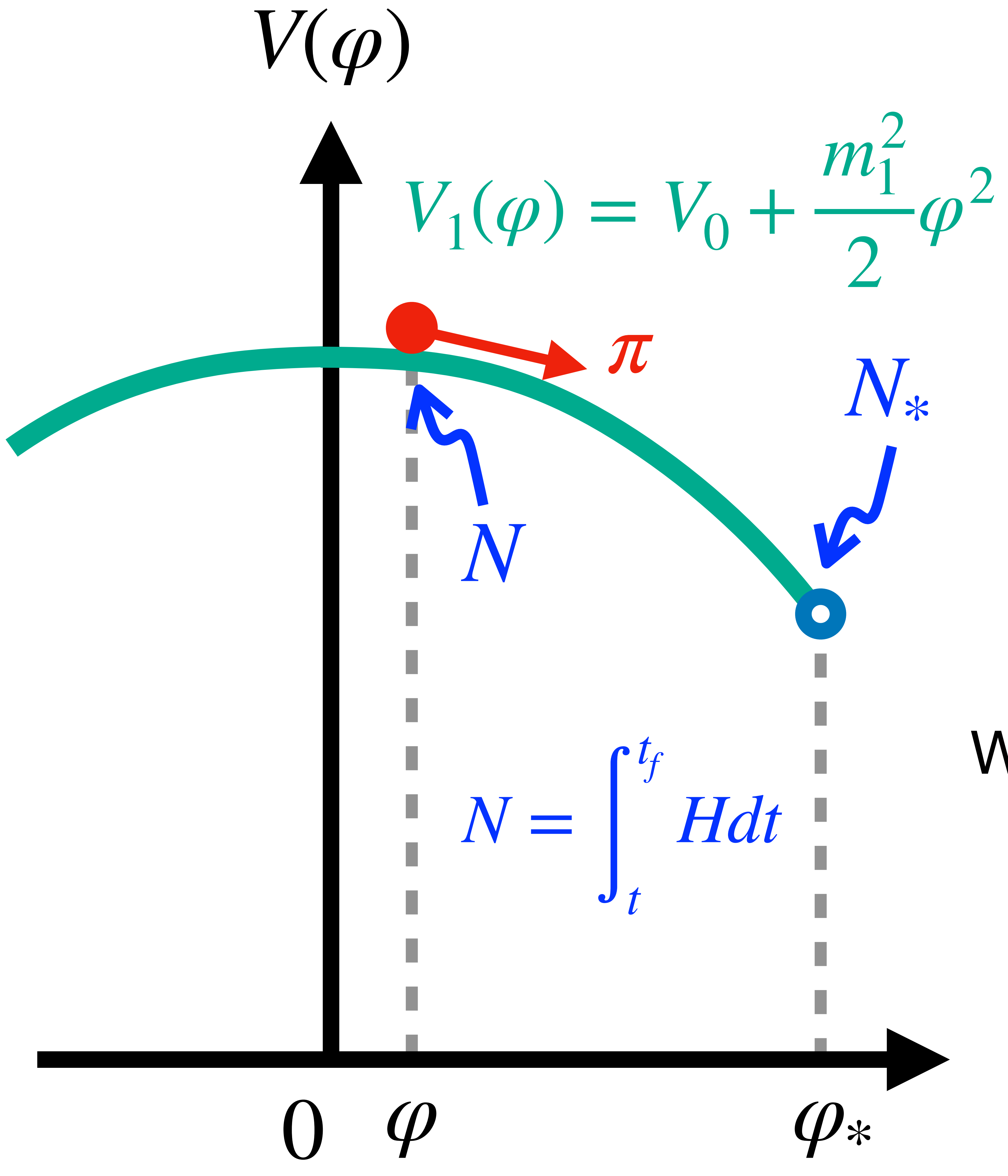
Curvaton scenario,
SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728

$\lambda = -6$

$$\mathcal{R} = -\frac{1}{6}\ln\left(1 - 6\mathcal{R}_{\rm G}\right)$$

Modulated reheating,
Shuichiro Yokoyama, in prep.

Logarithmic Duality



$$\frac{\partial^2 \varphi}{\partial^2 N} - 3\frac{\partial \varphi}{\partial N} + 3\eta_V \varphi = 0$$

$$\Longrightarrow \quad \varphi = c_+ e^{\lambda_+ N} + c_- e^{\lambda_- N}$$

$$\lambda_\pm = \frac{3 \pm \sqrt{9 - 12\eta_V}}{2} \qquad \eta_V = \frac{m_1^2}{3H^2}$$

We show that $\mathscr{R}$ can be expressed by two equivalent expressions:

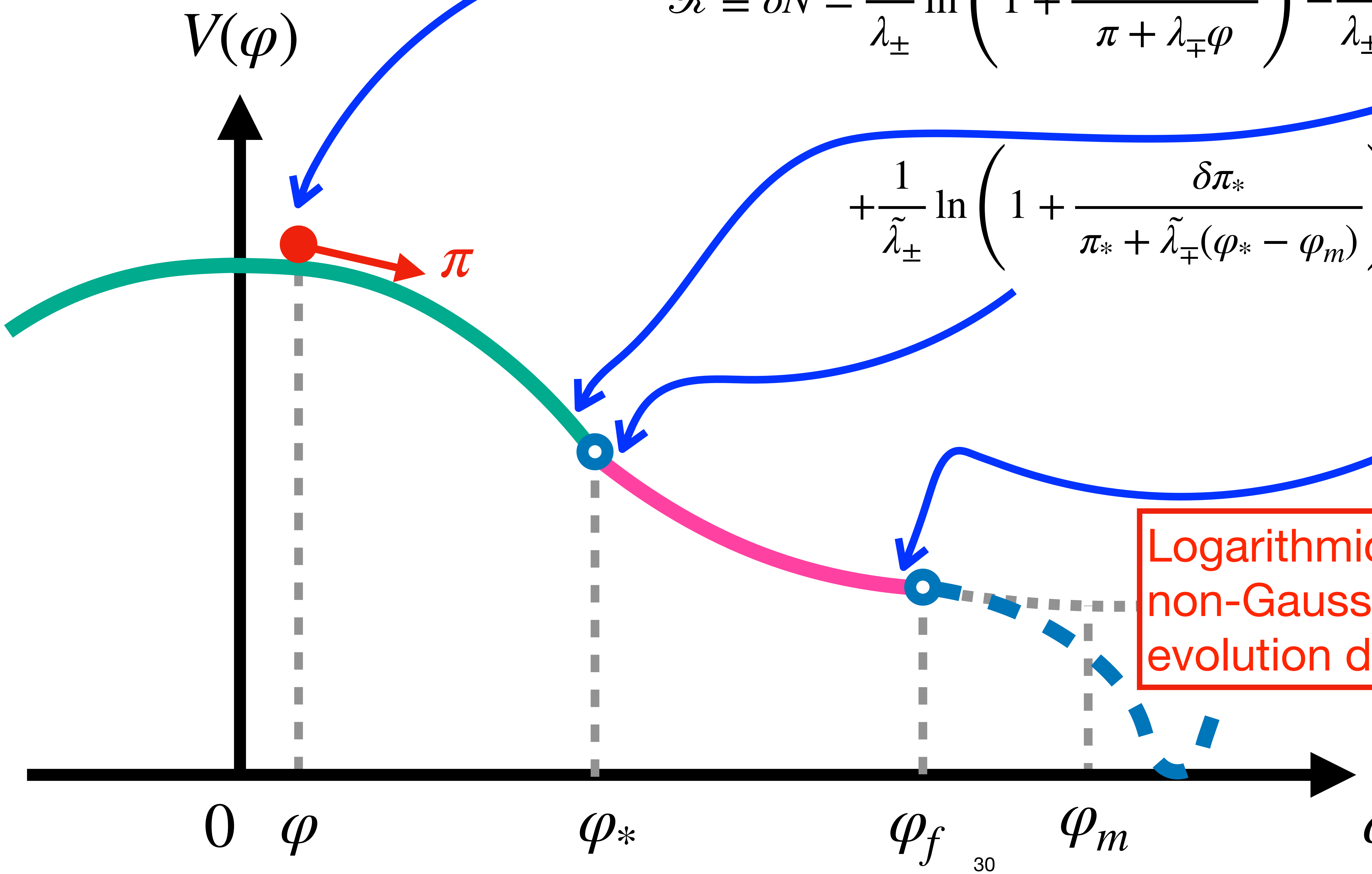
$$\mathscr{R} = \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi + \lambda_\mp \delta\varphi}{\pi + \lambda_\mp \varphi}\right) - \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_\mp \varphi_*}\right)$$

SP and Sasaki, 2211.13932

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$

$$+\frac{1}{\tilde{\lambda}_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\tilde{\lambda}_\mp(\varphi_*-\varphi_m)}\right) - \frac{1}{\tilde{\lambda}_\pm}\ln\left(1+\frac{\delta\pi_f}{\pi_f+\tilde{\lambda}_\mp(\varphi_f-\varphi_m)}\right)$$



Logarithmic relation, which leads to large non-Gaussianity, is generated when the evolution deviates from attractor solution.

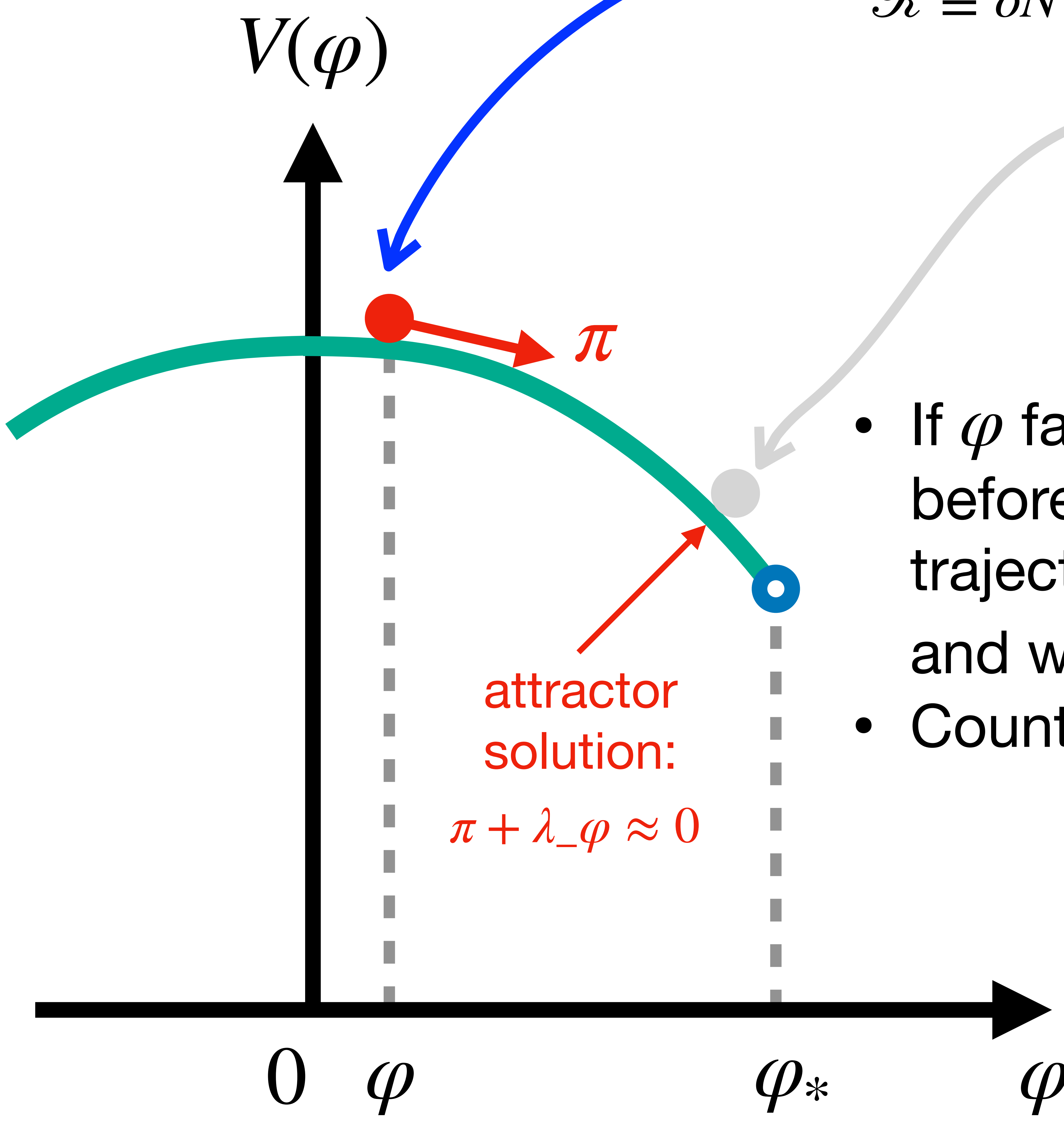
SP and Sasaki, 2211.13932

Application:

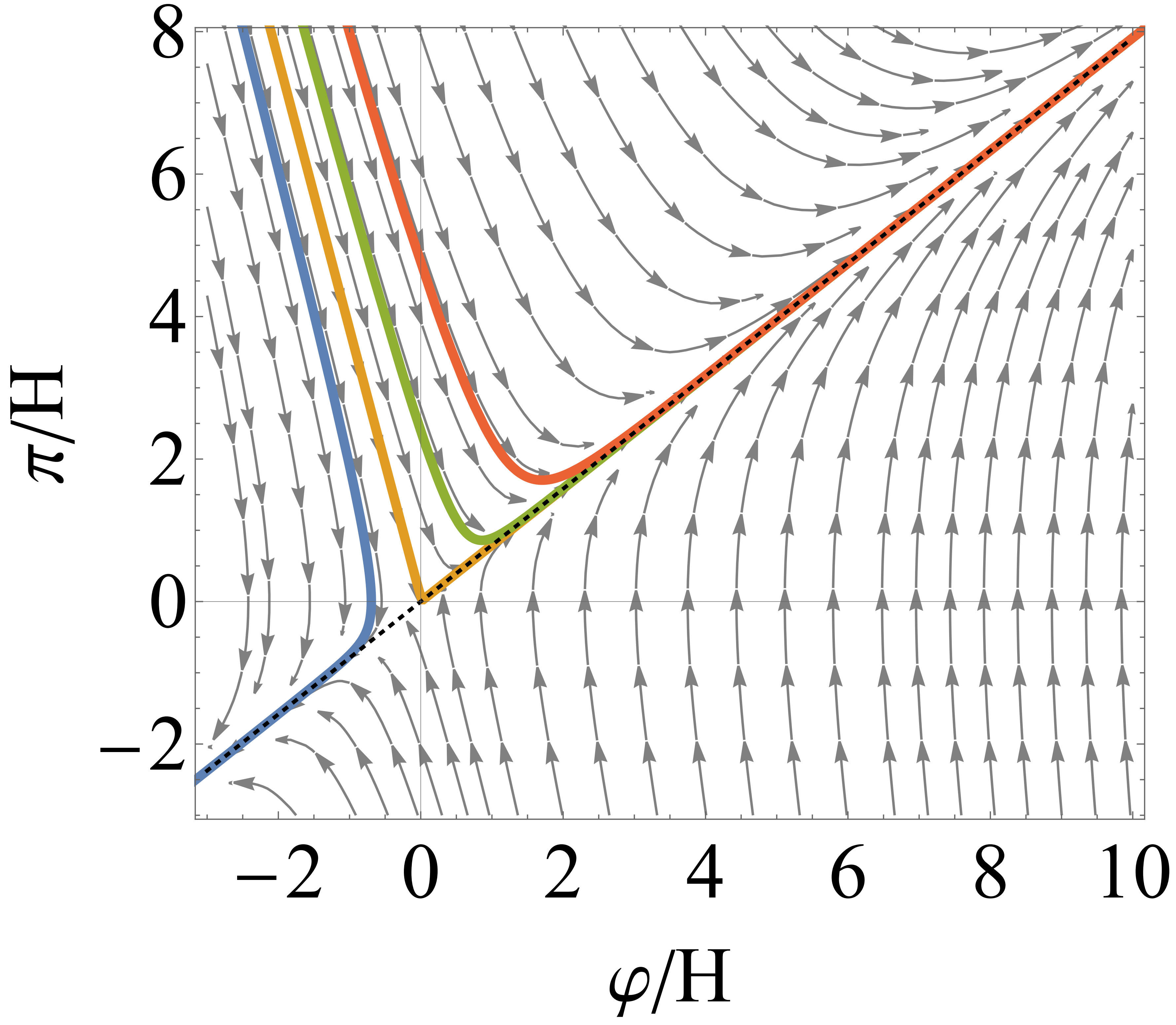
Constant-roll and Ultra-slow-roll inflation

Case 1： Bumpy potential

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right) - \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_+ \varphi_*}\right)$$



- If φ falls into the attractor before the boundary, its trajectory becomes unique and will not contribute to δN.
- Counterexample: USR

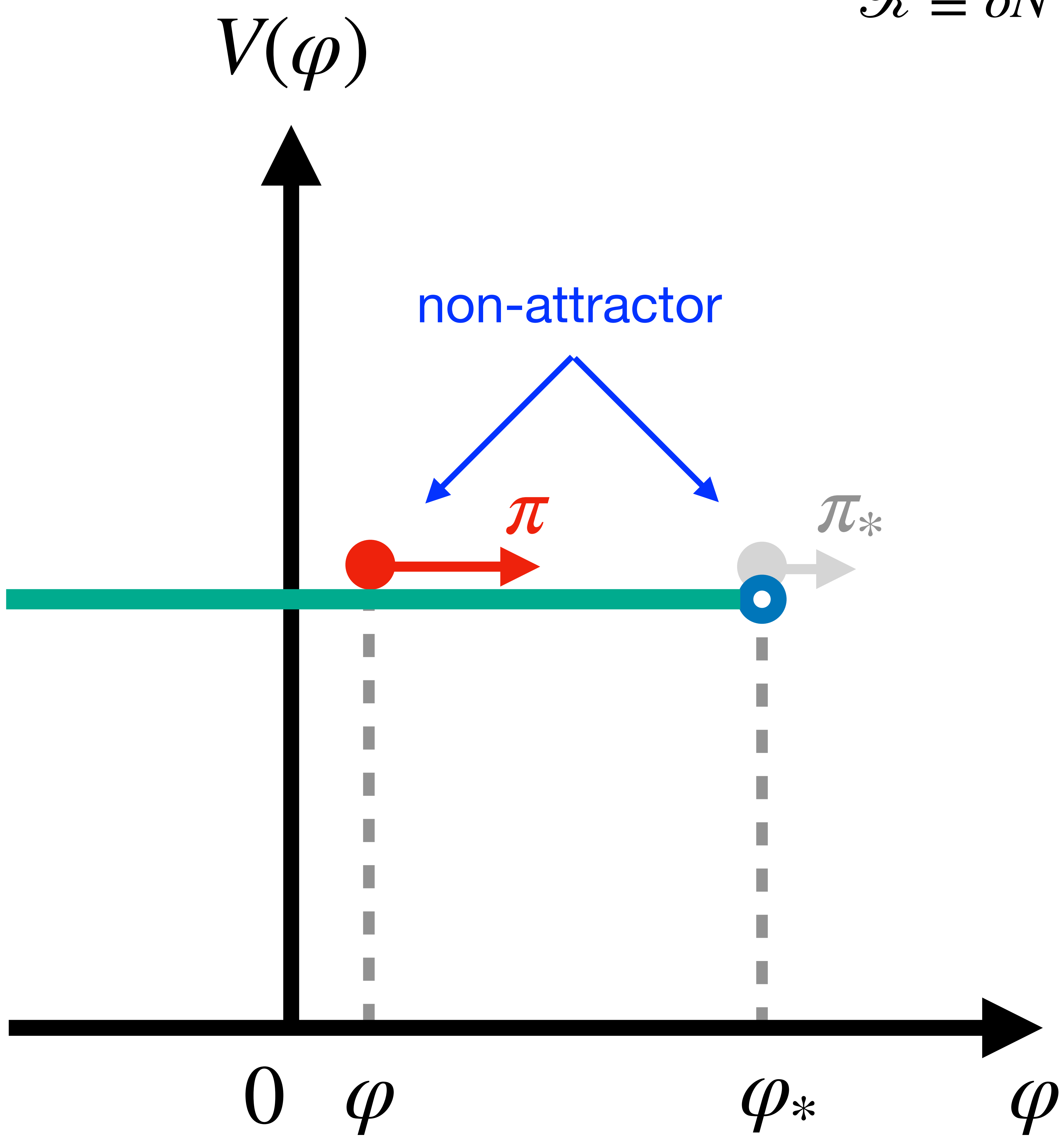


See also Atal et al, 1908.11357, 1905.13202

Case 2： USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$

$$\mathscr{R} \equiv \delta N = \frac{1}{3}\ln\left(1+\frac{\cancel{\delta\pi} + \cancel{\lambda_\mp}\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\cancel{\lambda_\mp}\varphi_*}\right)$$



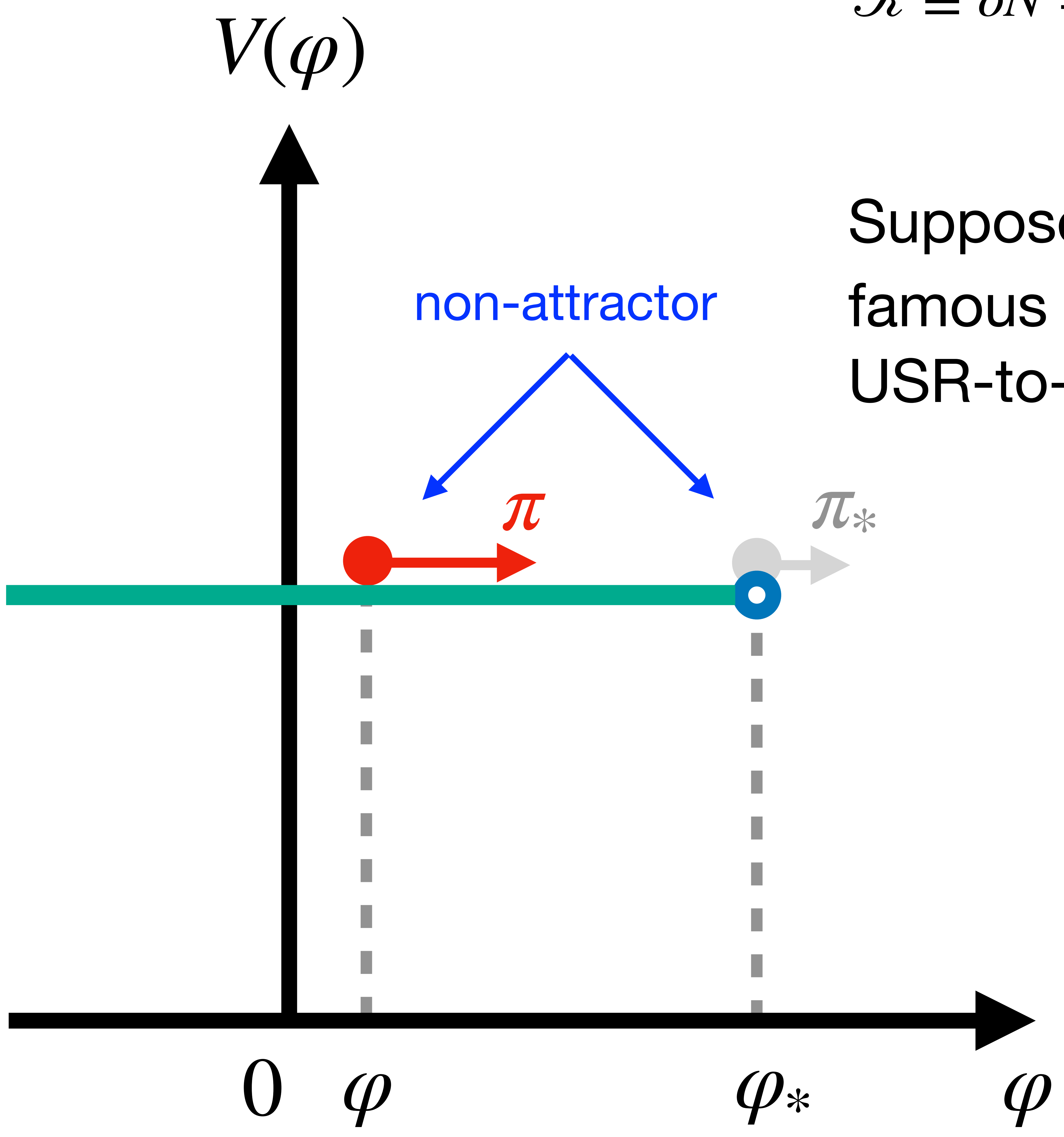
- If φ reaches the attractor solution before the boundary, it got stuck (classically), and quantum diffusion dominates. We must use stochastic approach to inflation.

Figueroa et al, 2012.06551
Pattison et al, 2101.05741
Rigopoulos & Wilkins, 2107.05317
Cruces & Germani, 2107.12735
Tada & Vennin, 2111.15280

Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$

$$\mathcal{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right)$$



Suppose inflation ends as the USR ends, it gives a famous result of $\mathbb{P}(\mathcal{R}) \propto \exp(-3\mathcal{R})$. However, the USR-to-SR transition should be considered.

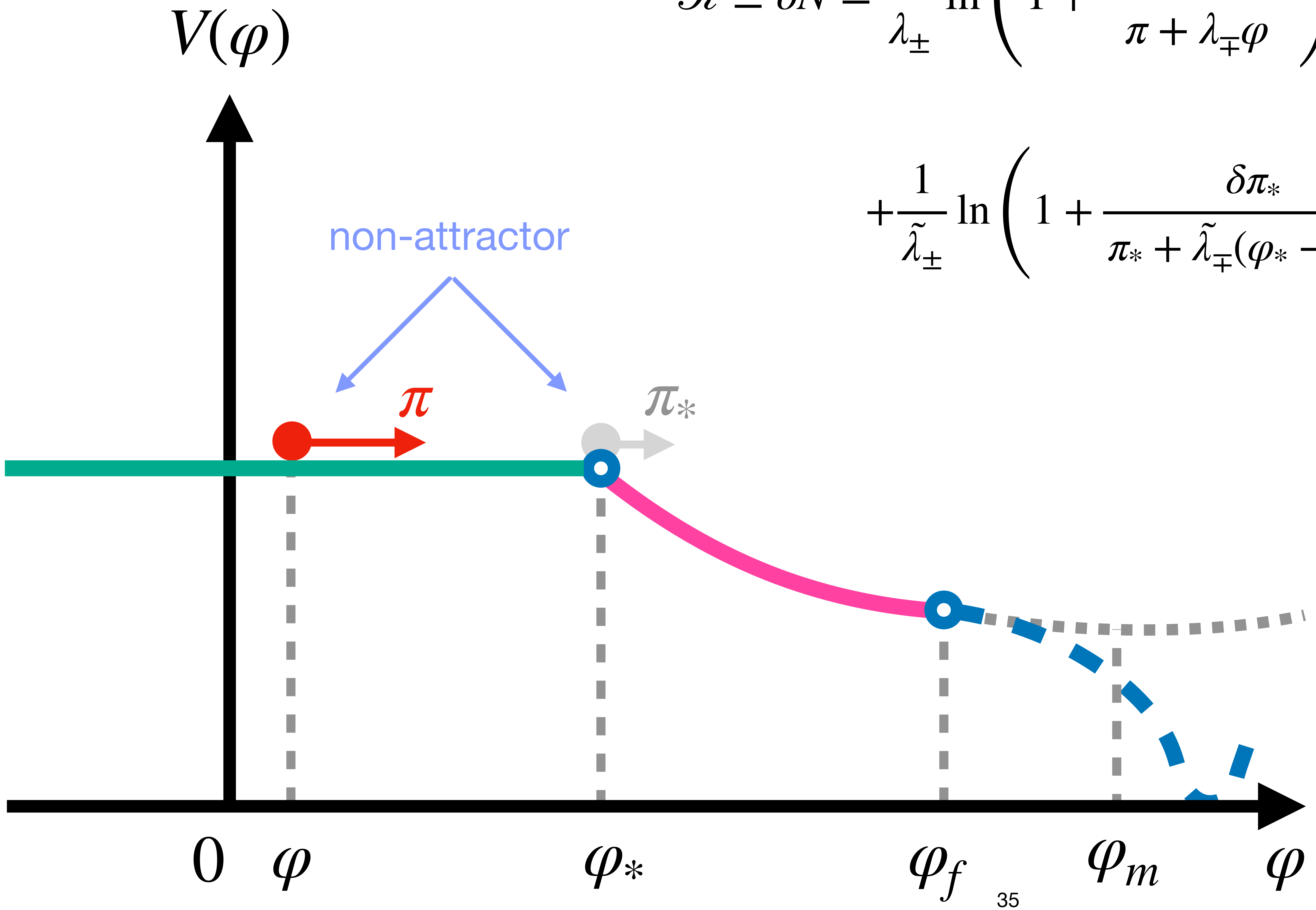
Namjoo et al., 1210.3692
Chen et al., 1308.5341

Case 2：USR

$$(\lambda_- = 0, \quad \lambda_+ = 3)$$
$$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$$

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi + \lambda_\mp \delta\varphi}{\pi + \lambda_\mp \varphi}\right) - \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_\pm \varphi_*}\right)$$

$$+ \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_\mp(\varphi_* - \varphi_m)}\right) - \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_f}{\pi_f + \tilde{\lambda}_\pm(\varphi_f - \varphi_m)}\right)$$

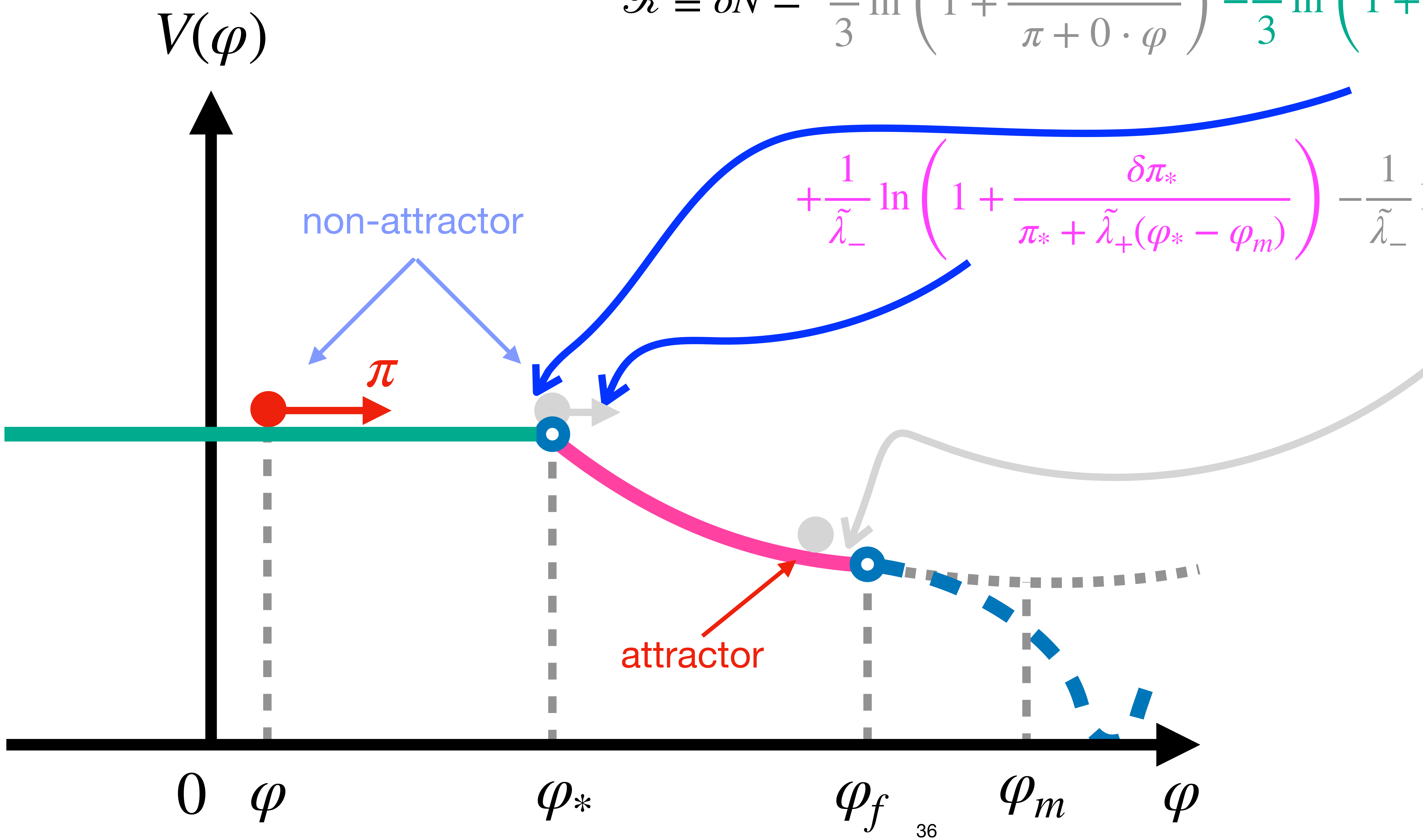


Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathcal{R} \equiv \delta N = \frac{1}{3}\ln\left(1+\frac{0+0\cdot\delta\varphi}{\pi+0\cdot\varphi}\right) - \frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

$$+\frac{1}{\tilde{\lambda}_-}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\tilde{\lambda}_+(\varphi_*-\varphi_m)}\right) - \frac{1}{\tilde{\lambda}_-}\ln\left(1+\frac{\delta\pi_f}{\pi_f+\tilde{\lambda}_+(\varphi_f-\varphi_m)}\right)$$



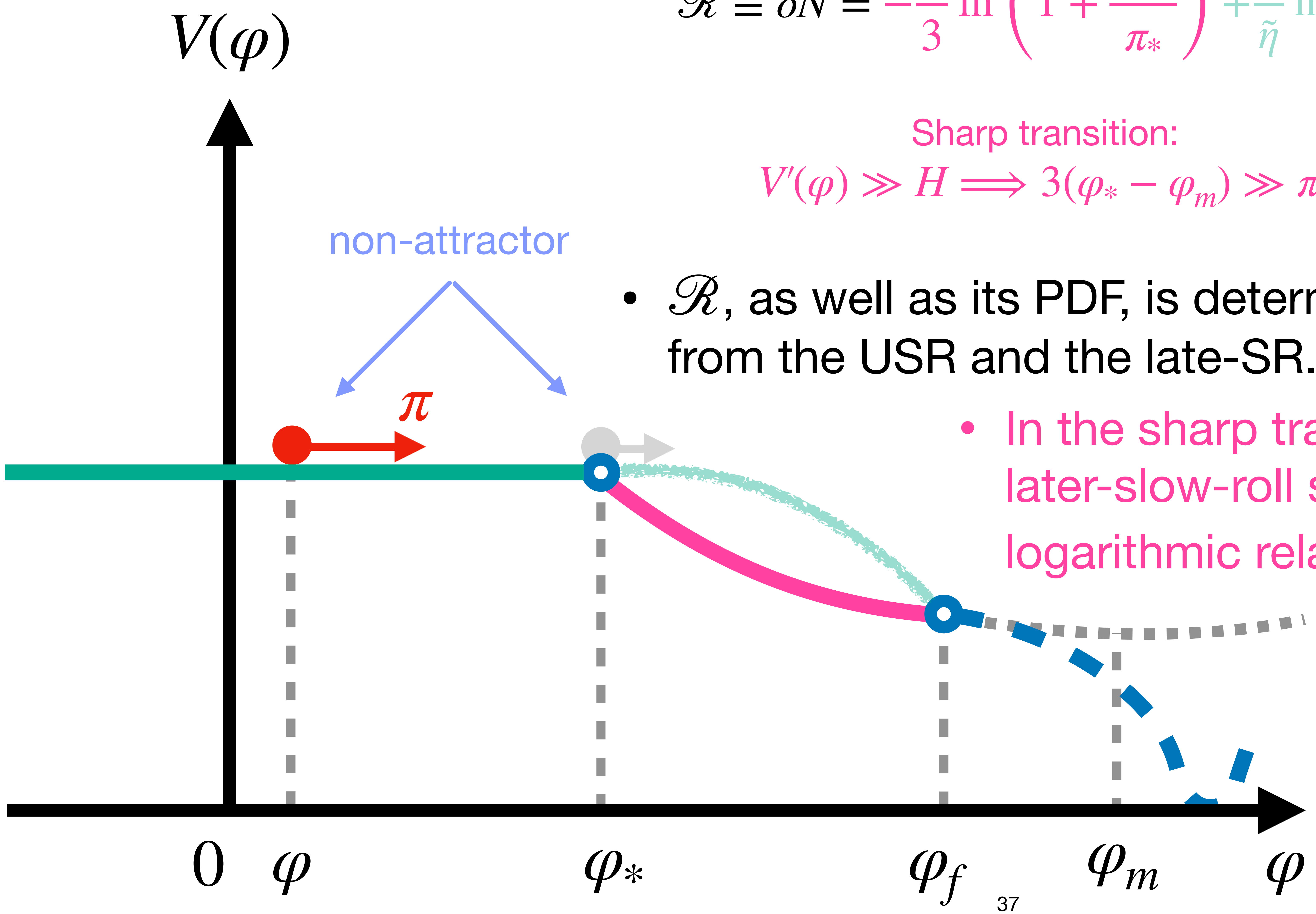
Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathscr{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

Sharp transition:
$V'(\varphi) \gg H \Longrightarrow 3(\varphi_* - \varphi_m) \gg \pi_*$

Smooth transition
$V'(\varphi) \ll H \Longrightarrow 3(\varphi_* - \varphi_m) \ll \pi_*$



- $\mathscr{R}$, as well as its PDF, is determined by the larger contribution from the USR and the late-SR.
  - In the sharp transition case, the contribution from later-slow-roll stage is negligible, thus the logarithmic relation of $\mathscr{R}(\delta\varphi)$ is preserved.

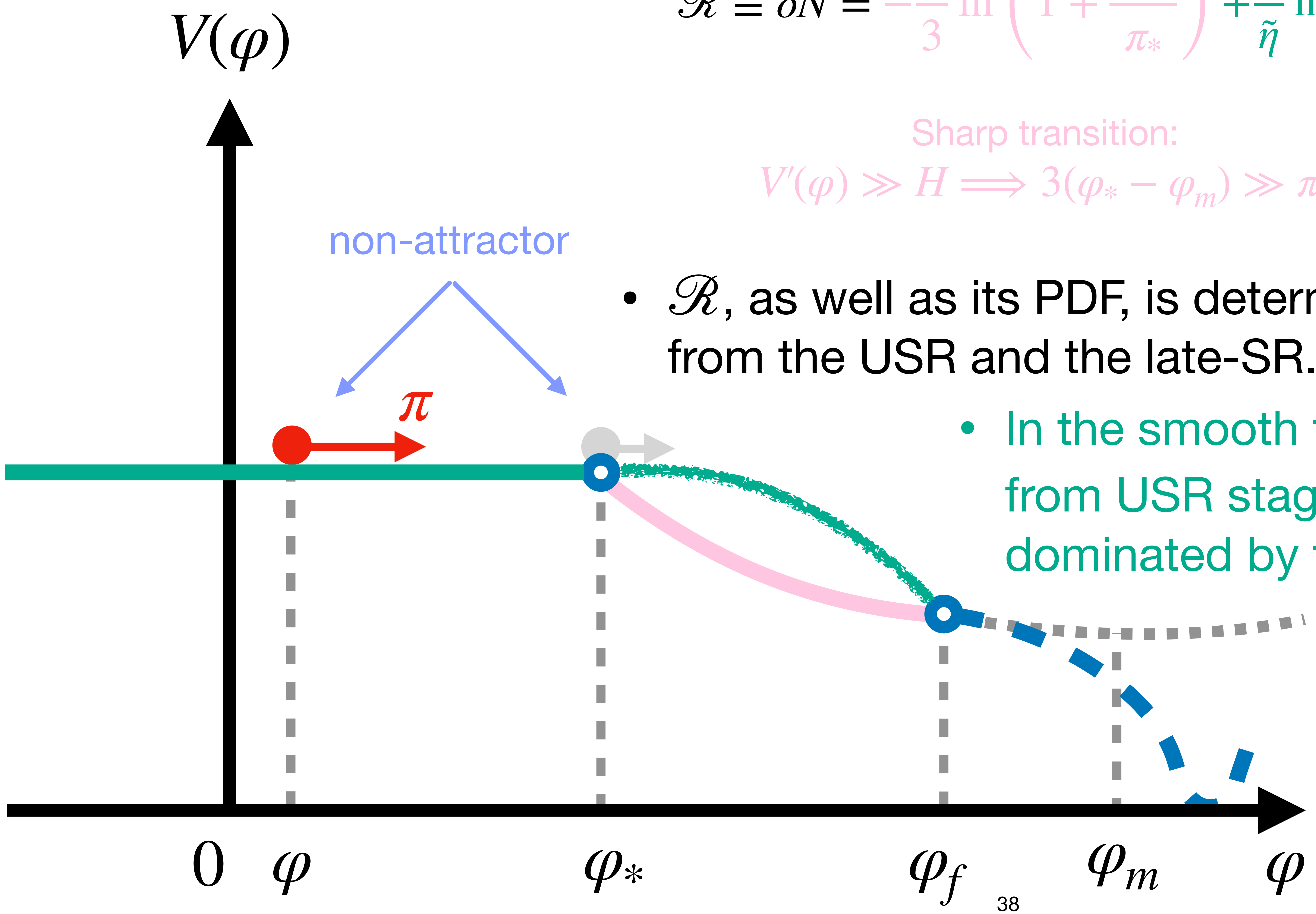
Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$
$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

$$\mathscr{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

Sharp transition:
$V'(\varphi) \gg H \Longrightarrow 3(\varphi_* - \varphi_m) \gg \pi_*$

Smooth transition
$V'(\varphi) \ll H \Longrightarrow 3(\varphi_* - \varphi_m) \ll \pi_*$



- $\mathscr{R}$, as well as its PDF, is determined by the larger contribution from the USR and the late-SR.
  - In the smooth transition case, the contribution from USR stage is negligible, and $\mathscr{R}(\delta\varphi)$ is dominated by the slow-roll part.

Case 2: USR

$(\lambda_- = 0, \quad \lambda_+ = 3)$

$(\tilde{\lambda}_- = \tilde{\eta}, \quad \tilde{\lambda}_+ = 3 - \tilde{\eta})$

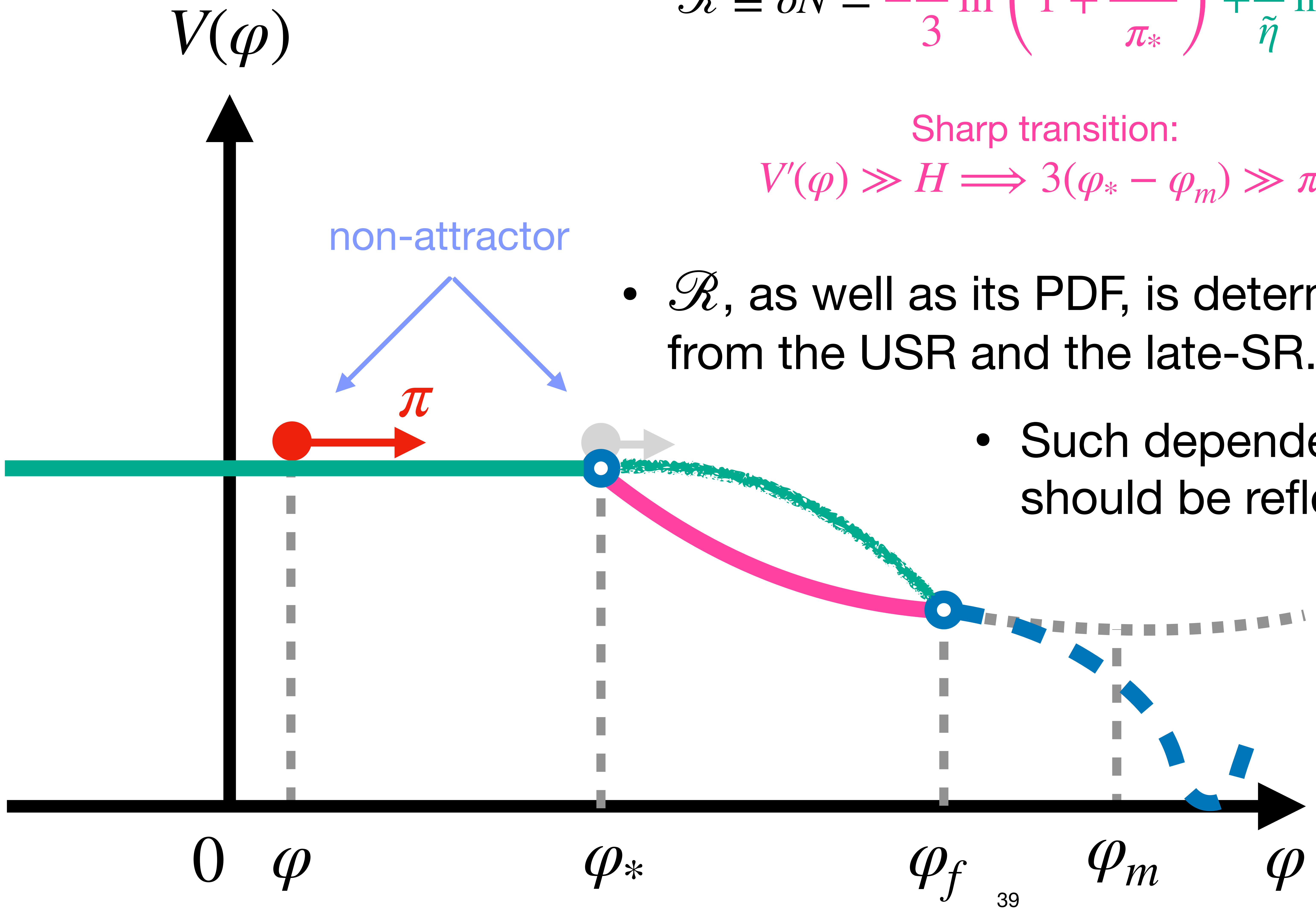
$$\mathscr{R} \equiv \delta N = -\frac{1}{3}\ln\left(1 + \frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\eta}}\ln\left(1 + \frac{\delta\pi_*}{\pi_* + (3-\tilde{\eta})(\varphi_* - \varphi_m)}\right)$$

Sharp transition:

$$V'(\varphi) \gg H \Longrightarrow 3(\varphi_* - \varphi_m) \gg \pi_*$$

Smooth transition

$$V'(\varphi) \ll H \Longrightarrow 3(\varphi_* - \varphi_m) \ll \pi_*$$



- $\mathscr{R}$, as well as its PDF, is determined by the larger contribution from the USR and the late-SR.

- Such dependence on the boundary condition should be reflected in the stochastic approach.

Pattison et al., 2101.05741
Cruces, SP, Sasaki, in prep.

- Sharp transition will make the separate universe approach (thus δN formalism) invalid transiently.

Domenech et al., 2309.05750
Jackson et al., 2311.03281

Probability Distribution Function

For the USR case we use the dual relation: $\mathscr{R} \equiv \delta N = -\frac{1}{3}\ln\left(1+\frac{3\delta\varphi}{\pi_*}\right)$

$$\Downarrow \quad \mathbb{P}(\mathscr{R})d\mathscr{R} = \mathbb{P}(\delta\varphi)d\delta\varphi$$

$\mathbb{P}(\delta\varphi)$ → Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$\mathbb{P}(\mathscr{R}) = \frac{e^{-3\mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}}\pi_* \exp\left[-\frac{\pi_*^2}{18\sigma_{\delta\varphi}^2}\left(e^{-3\mathscr{R}}-1\right)^2\right]$$

$$\downarrow \quad \mathscr{R} \sim \mathcal{O}(1)$$

$$\mathbb{P}(\mathscr{R}) \sim e^{-3\mathscr{R}}$$

exponential tail

Biagetti et al, 2105.07810
Pattison ett al, 2101.05741
SP and Sasaki, 2211.13932

Probability Distribution Function

For the simplest single-logarithm case: $\mathcal{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right)$

$$\Downarrow \quad \mathbb{P}(\mathcal{R})d\mathcal{R} = \mathbb{P}(\delta\varphi)d\delta\varphi$$

$\mathbb{P}(\delta\varphi)$ → Gaussian PDF with variance $\sigma^2_{\delta\varphi}$

$$\mathbb{P}(\mathcal{R}) = \frac{e^{\lambda_- \mathcal{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma^2_{\delta\varphi}}\left(e^{\lambda_- \mathcal{R}} - 1\right)^2\right]$$

$\lambda_- < 0$, $\mathcal{R} \sim \mathcal{O}(1)$:

$\mathbb{P}(\mathcal{R}) \sim e^{\lambda_- \mathcal{R}}$

exponential tail

$\lambda_- > 0$, $\mathcal{R} \sim \mathcal{O}(1)$:

$\mathbb{P}(\mathcal{R}) \sim \exp\left(-c^2 e^{2\lambda_- \mathcal{R}}\right)$

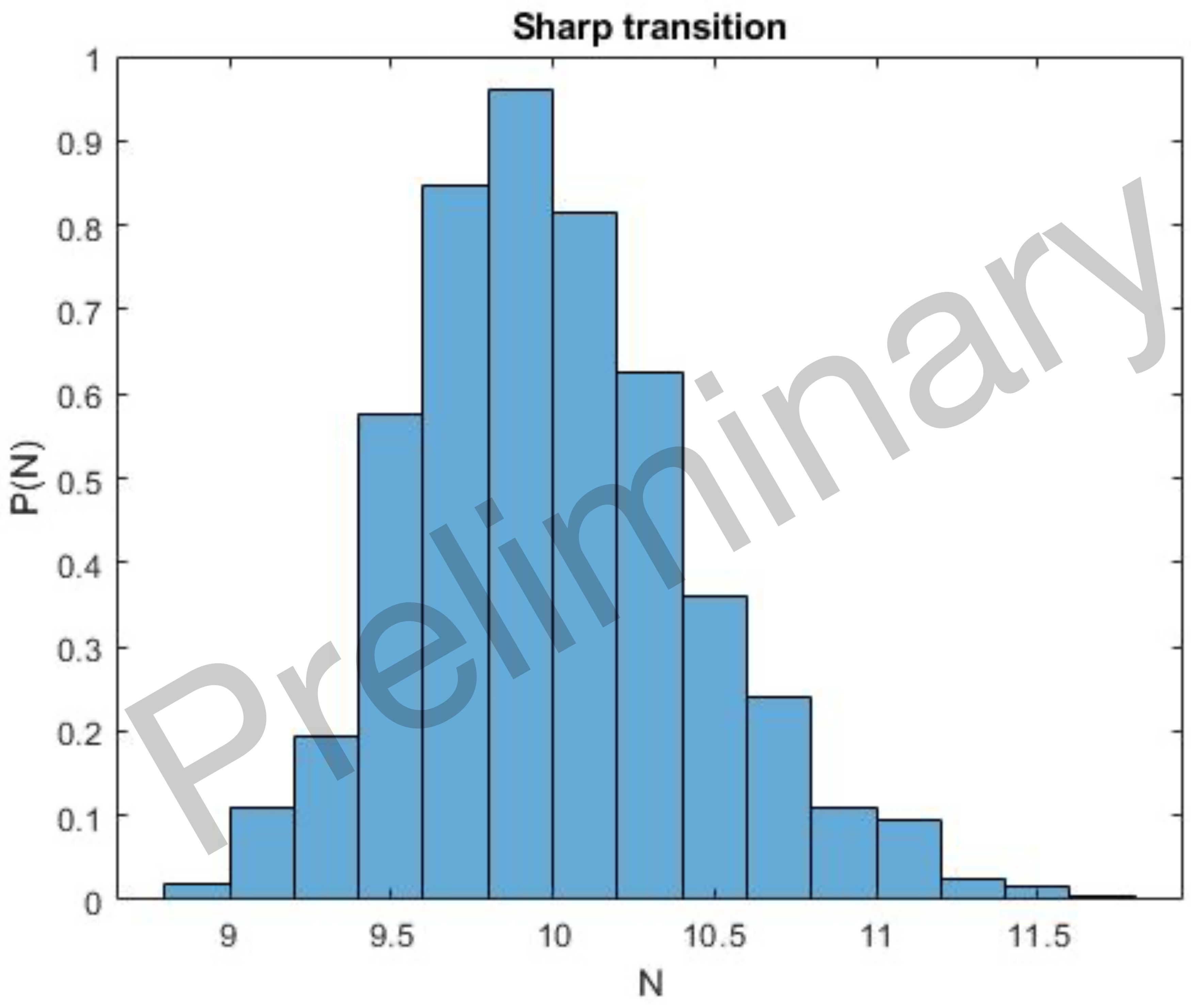
Gumbel-distribution-like tail

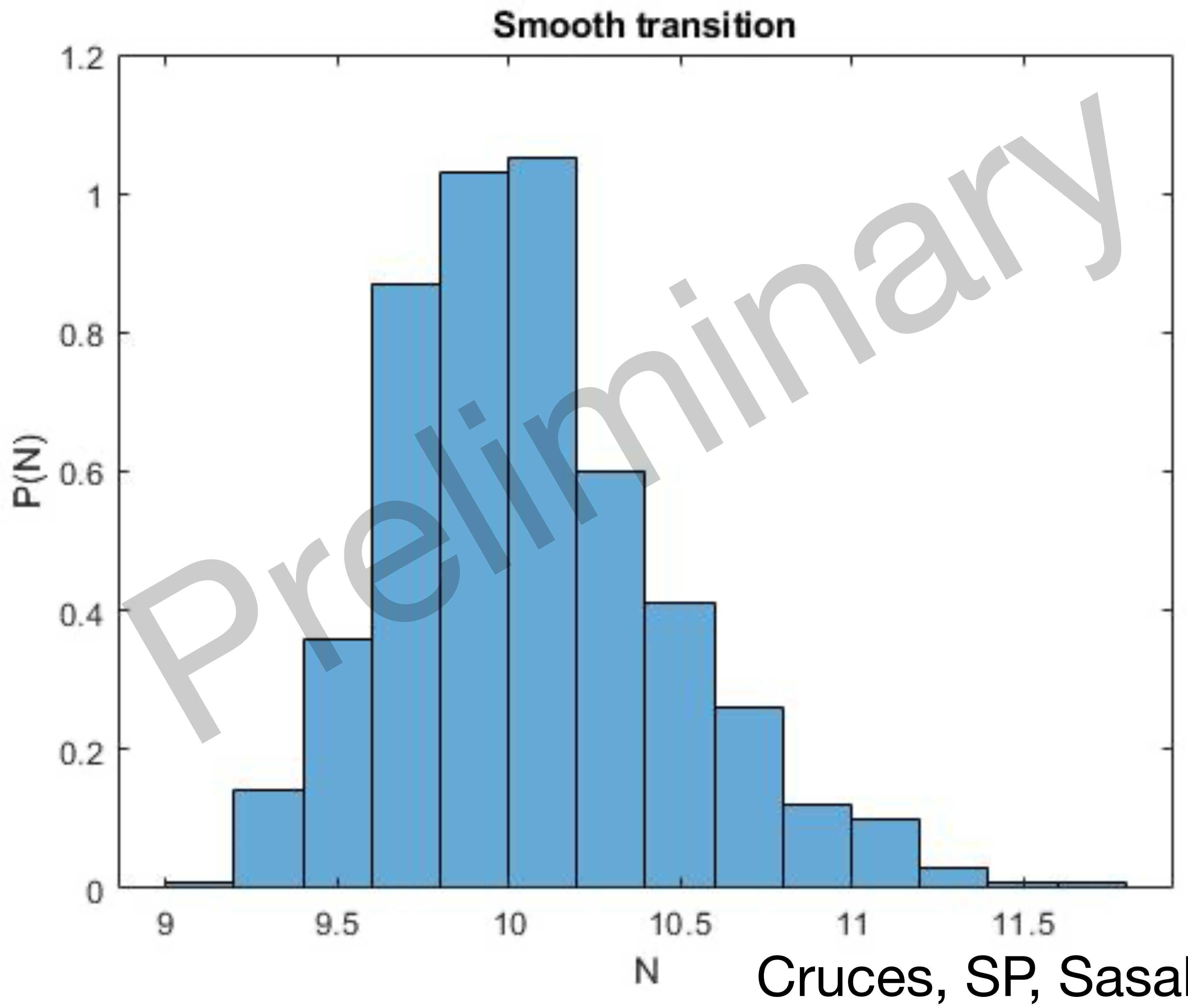
SP and Sasaki, 2211.13932

Probability Distribution Function

For a general case: $$\mathscr{R} = -\frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right) + \frac{1}{\tilde{\lambda}_-}\ln\left(1+\frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_+(\varphi_* - \varphi_m)}\right) \qquad \left(\tilde{\lambda}_- = -\frac{1}{2}, \quad \tilde{\lambda}_+ = \frac{7}{2}\right)$$

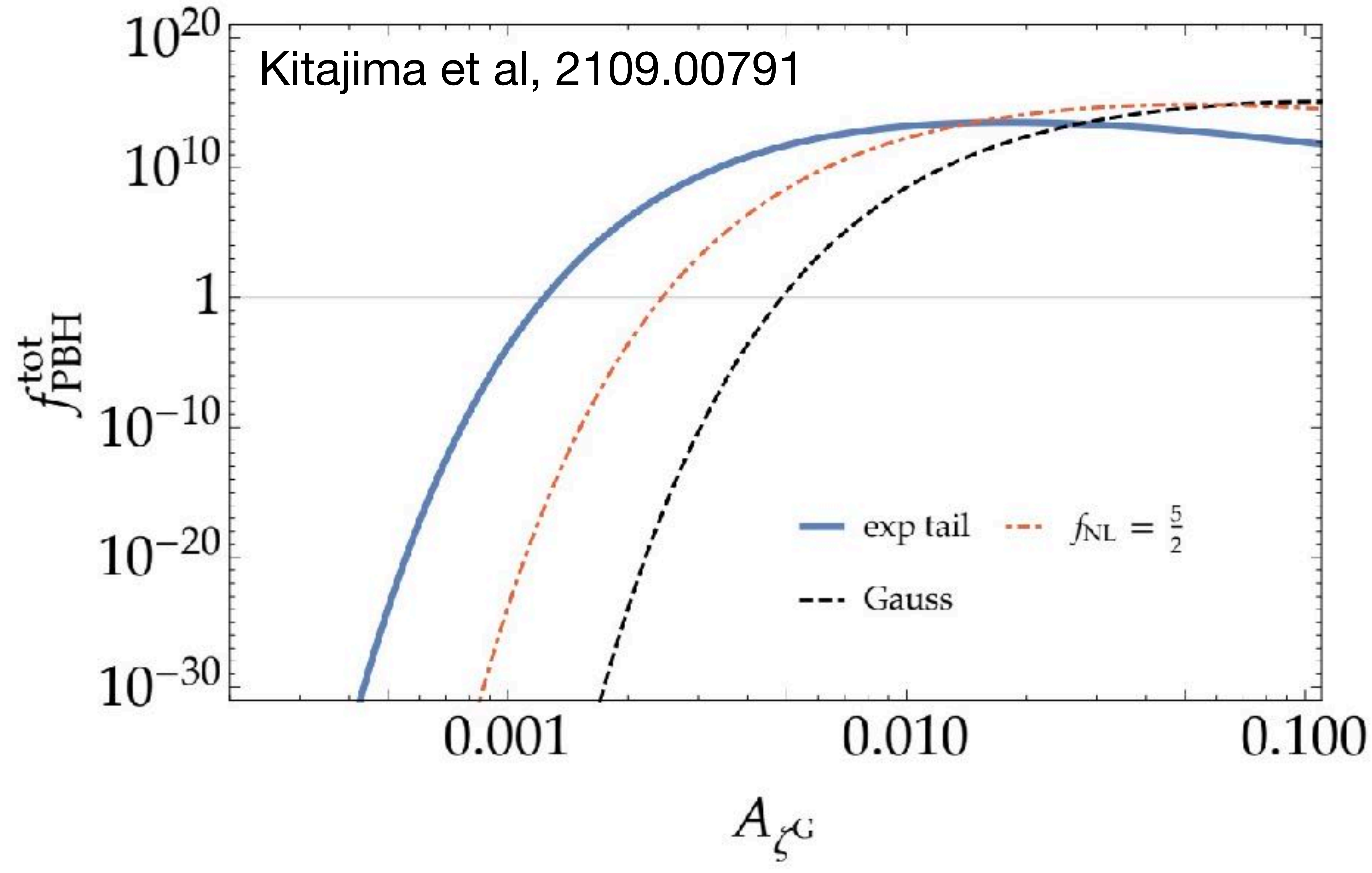
- It shows that smooth transition could be even "more non-Gaussian", depending on the value of $\tilde{\lambda}_-$.

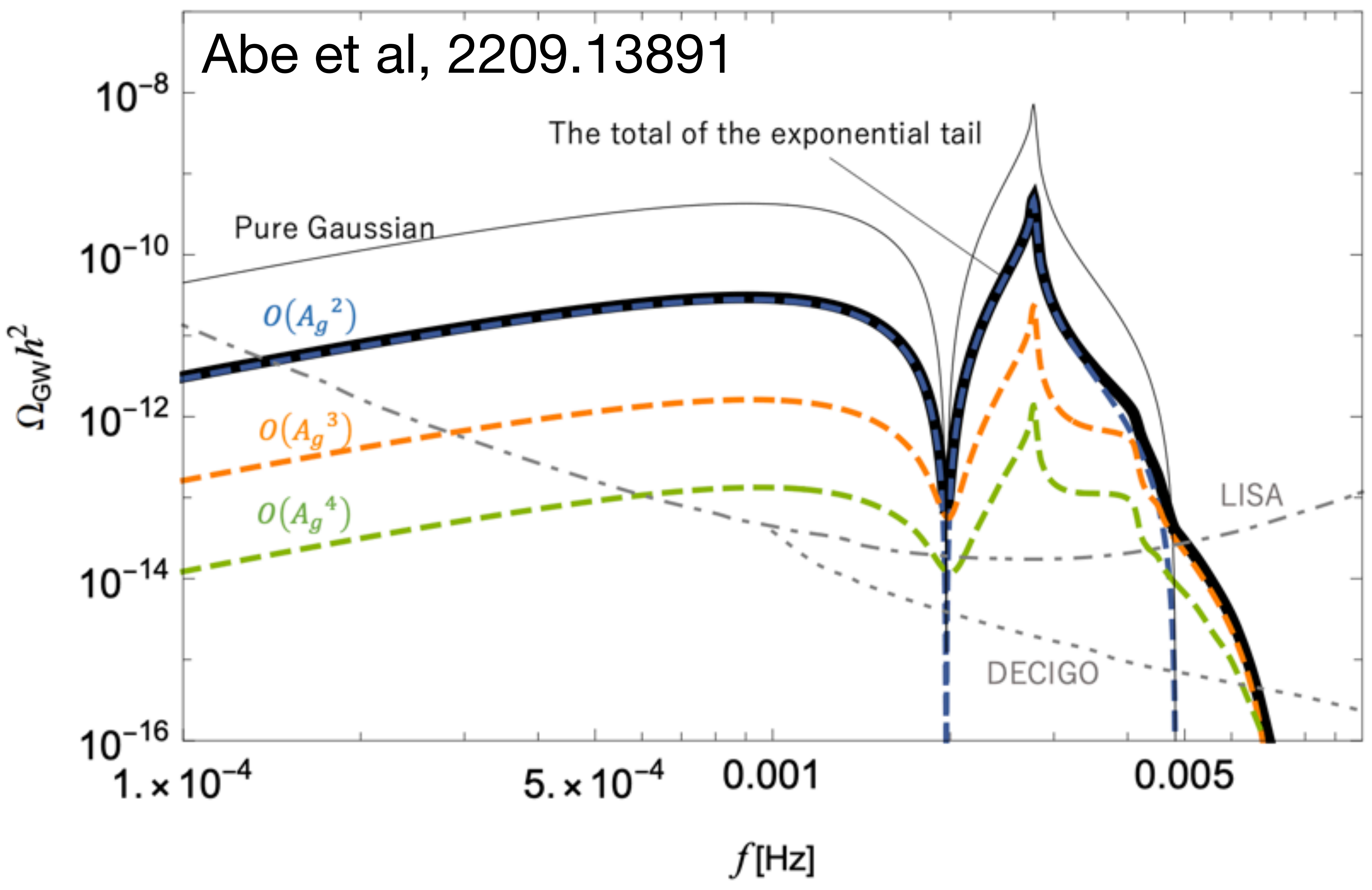




Cruces, SP, Sasaki, in prep.

PBH and IGW with NG





SP, Sasaki, Takhistov, Jianing Wang, in prep.

Jianing's Talk 1130-B3-1

Discussion

- Once $V(\varphi)$ deviates from quadratic, we may have heavier or lighter tails than the exponential tail $P(\mathscr{R}) \propto \exp(-\lambda |\mathscr{R}|^p)$. See Nakama, Suyama, Yokoyama, 1609.02245; Namjoo et al 2112.04520, 2305.19257; Creminelli, Yingcharoenrat et al 2103.09244.

- We did not consider the case when $V(\varphi)$ itself has discontinuity. See Cai, Sasaki et al 2112.13836; Fujita, Sasaki et al 2305.18140.

- The PDF of USR should depend on how USR ends, which should be reflected in the boundary conditions of the stochastic approach of inflation. See Pattison et al., 2101.05741; Tada and Vennin, 2111.15280; Cruces, SP, Sasaki, in preparation.

- What is the PDF of more complicated (or intermediate) cases when contributions from two stages are comparable thus $\mathscr{R}(\delta\varphi)$ can not be written as a single logarithm?

Summary

- The simplest Press-Schechter ignores non-Gaussianities of different origins, which greatly(mildly) enhance/suppress the PBH abundance (IGW spectrum) when NG is positive/negative.

- Primordial non-Gaussianity in $\mathscr{R}(\delta\varphi)$ originates from the non-attractor evolution. The final $\mathscr{R}(\delta\varphi)$ is a sum of all the stages. If $\mathscr{R}(\delta\varphi)$ is dominated by one stage, $\mathbb{P}(\mathscr{R})$ displays an exponential tail or a Gumbel-like (double exponential suppression) tail, depending on the signature of $V''(\varphi)$.

- When $|f_{\rm NL}| \sim \mathcal{O}(1)$, all the NG effect must be taken appropriately to calculate the PBH abundance. This is necessary when interpreting nHz GW as the IGW.

Thank you for your attendance!

どうもありがとうございます！